Frequency Allocation Problems for Linear Cellular Networks *

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Abstract. We study the online frequency allocation problem for wireless linear (highway) cellular networks, where the geographical coverage area is divided into cells aligned in a line. Calls arrive over time and are served by assigning frequencies to them, and no two calls emanating from the same cell or neighboring cells are assigned the same frequency. The objective is to minimize the span of frequencies used.
In this paper we consider the problem with or without the assumption that calls have infinite duration. If there is the assumption, we propose an algorithm with absolute competitive ratio of 3/2 and asymptotic competitive ratio of 1.382. The lower bounds are also given: the absolute one is 3/2 and the asymptotic one is 4/3. Thus, our algorithm with absolute ratio of 3/2 is best possible. We also prove that the Greedy algorithm is 3/2-competitive in both the absolute and asymptotic cases. For the problem without the assumption, i.e. calls may terminate at arbitrary time, we give the lower bounds for the competitive ratios: the absolute one is 5/3 and the asymptotic one is 14/9. We propose an optimal online algorithm with both competitive ratio of 5/3, which is better than the Greedy algorithm, with both competitive ratios 2.

1 Introduction

Reducing channel interference and using frequencies effectively are fundamental problems in wireless networks based on Frequency Division Multiplexing (FDM) technology. In FDM networks, service areas are usually divided into cellular regions or *hexagonal cells* [7], each containing one base station. Base stations can allocate radio frequencies to serve the phone calls in their cells. The allocation strategy is to choose different frequencies for calls in the same cell or in the neighboring cells, so as to avoid interference.

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We consider the problems of *online frequency allocation* in *linear (or highway)* cellular networks where the cells are aligned in a line as shown in Fig. 1. Linear cellular networks can be used to cover the traffic on highways or long strips of busy metropolitan areas. There are many studies on using frequencies effectively so as to minimize interference and to reduce call blocking in linear networks [1, 2, 6, 8]. In this paper we study the performances of different strategies, which are to minimize the span of frequencies used to serve all calls without interference.

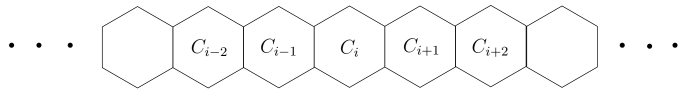

Fig. 1. Linear cellular network

A formal definition of our problem is described as follows. Given a linear cellular network, in which a sequence σ of calls arrive over time, where $\sigma = (C_{t_1}, C_{t_2}, \ldots, C_{t_k}, \ldots,)$ and C_{t_k} represents the cell from which the k-th call emanates. Each call C_{t_k} must be assigned upon its arrival, without information about future calls $\{C_{t_i} | i > k\}$, a frequency from the integer set $Z^+ = \{1, 2, \ldots\}$ of available frequencies, that is different from those of other calls in the same cell or neighboring cells. Let $\mathcal{A}(C_{t_k}) \in Z^+$ denote the integer frequency assigned to the k-th call. Then $\mathcal{A}(C_{t_k}) \neq \mathcal{A}(C_{t_i})$, where $i < k$ and C_{t_i} is adjacent to C_{t_k} or the same as C_{t_k}. The integer frequency once assigned to a call cannot be changed during the survival of this call. The *online frequency allocation problem for linear cellular network* (FAL for short) is to minimize the maximum assigned frequency, i.e., $\max\{\mathcal{A}(C_{t_k}) | k = 1, 2, \ldots, n\}$. If all the information of C_{t_k} is known in advance, we call this problem *off-line frequency allocation problem.* In this paper, we focus on the online version of FAL.

Two models of online frequency allocation problems will be investigated. The first model is that all calls have infinite duration [4]. We call this model *frequency allocation without deletion.* The second model is that each call may terminate at arbitrary time, i.e., each call is characterized by two parameters: arrival time and termination time. However, the termination time is not known even when the call arrives online. We call this model *frequency allocation with deletion.*

Performance Measures. We use competitive analysis [3] to measure the performance of online algorithms. For any sequence σ of calls, let σ_t denote the subsequence of calls served up to and at time t. Let $\mathcal{A}(\sigma_t)$ denote the cost of an online algorithm $\mathcal{A}$, i.e., the span of frequencies used by $\mathcal{A}$ at time t, and $\mathcal{O}(\sigma_t)$ the cost of the optimal off-line algorithm, which has the knowledge of the whole sequence σ in advance.

Let $\mathcal{A}(\sigma) = \max_t \mathcal{A}(\sigma_t)$ and $\mathcal{O}(\sigma) = \max_t \mathcal{O}(\sigma_t)$. The *(absolute) competitive ratio* of $\mathcal{A}$ is defined as $R_{\mathcal{A}} = \sup_\sigma \mathcal{A}(\sigma)/\mathcal{O}(\sigma)$. Meanwhile, when the number of calls emanating from the cells is large, the *asymptotic competitive ratio* of $\mathcal{A}$ is defined as

$$R_{\mathcal{A}}^{\infty} = \limsup_{n\to\infty} \max_{\sigma} \left\{ \frac{\mathcal{A}(\sigma)}{\mathcal{O}(\sigma)} \mid \mathcal{O}(\sigma) = n \right\}.$$

Clearly, for any online algorithm $\mathcal{A}$, we have $R^{\infty}_{\mathcal{A}} \le R_{\mathcal{A}}$.

Related and Our Contributions. To our best knowledge, this is the first study on the online frequency allocation problem in linear cellular networks with the objective to minimize the span of frequencies used. It is easy to check that the off-line version can be solved in polynomial time. However, in many practical scenarios, the information of calls is not completely known until they arrive. The online problem is more suitable to model the mobile telephone networks problem.

A simple strategy for the online FAL problem is by fixed allocation assignment [7], in which cells are partitioned into independent sets with no neighboring cells in the same set. Each set is assigned a separate set of frequencies. The fixed allocation assignment algorithm gives an easy upper bound of 2 for the online FAL problem.

Another intuitive approach is by the *greedy algorithm* (Greedy) which assigns the minimum available frequency to a new call such that the call does not interfere with calls of the same or neighboring cells. We show that, Greedy is 3/2-competitive in the *without deletion* model, and 2-competitive in the *with deletion* model.

In this paper, new algorithms are proposed for both models. In the *without deletion* model, we present the algorithm HYBRID, which combines the idea of Greedy and fixed allocation strategy, and yields the absolute and asymptotic competitive ratios of 3/2 and 1.382, respectively. Contrasting with the lower bounds shown, 3/2 for the absolute case and 4/3 for the asymptotic case, HYBRID is also best possible in the absolute case and better than Greedy in the asymptotic case.

In the *with deletion* model, we propose the algorithm BORROW with both absolute and asymptotic competitive ratios 5/3. We also prove the lower bounds, which is 5/3 for the absolute case and 14/9 for the asymptotic case. Thus, BORROW is best possible in the absolute case, and also better than Greedy in the asymptotic case.

The rest of this paper is organized as follows. In Section 2, we analyze the performance of Greedy. Section 3 and Section 4 study respectively the *without deletion* and the *with deletion* models, in which upper and lower bounds are presented. Owing to the space limitation, the proofs of Lemma 1 and part of the proofs for Theorems 2 and 5 are omitted but they will be given in the full paper.

2 The Greedy Algorithm

We first consider the *without deletion* model and prove an upper bound of 3/2 for the Greedy. The proved ratio applies to both absolute and asymptotic cases. Greedy is in fact optimal in the absolute case because no online algorithm can achieve an absolute competitive ratio less than 3/2 (Theorem 6) in that case. Then, we show that Greedy is 2-competitive in both absolute and asymptotic cases of the *with deletion* model, and that is the best that Greedy can do.

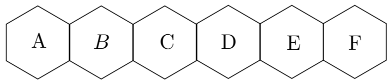

Fig. 2. A line cellular network with cells A, B, C, D, E and F.

Theorem 1. *In the without deletion model, the competitive ratio of Greedy for frequency allocation problem in linear cellular network is 3/2.*

Proof. Consider the network in Fig. 2. Suppose Greedy assigns the highest frequency to a call from cell C, and no more calls arrive after that. Let B and D be the left and right neighboring cells of C. Let f_X denote the set of frequencies used in cell X at the time when the highest frequency is assigned.

By the definition of Greedy, when the highest frequency, say h, is assigned to a call in C, the frequencies from 1 to $h-1$ must have been assigned to calls of C or its neighboring cells B and D. Thus, the span of frequencies used by Greedy is $h = |f_B \cup f_C \cup f_D|$.

Without loss of generality, assume the highest frequency among B and D appears in B. Since f_C and f_D cannot have common frequencies, those frequencies in $f_D - f_B$ must all appear in A. Therefore, $|f_A \cup f_B| \geq |f_B \cup f_D|$.

It is clear that the optimal span of frequencies used, say s^*, must be at least the maximum number of calls (frequencies used) from any two adjacent cells. Thus, we have $s^* \geq \max\{|f_A \cup f_B|, |f_B \cup f_C|, |f_C \cup f_D|\} \geq \max\{|f_B \cup f_D|, |f_B \cup f_C|, |f_C \cup f_D|\}$. Therefore, the competitive ratio of Greedy is at most

$$\frac{|f_B \cup f_C \cup f_D|}{\max\{|f_B \cup f_D|, |f_B \cup f_C|, |f_C \cup f_D|\}} \leq 3/2.$$

□

Theorem 2. *In the with deletion model, the upper and lower bounds of the competitive ratio of Greedy are both* 2 *for the online frequency allocation problem in linear cellular networks.*

Proof. The upper bound proof is simple. Consider the network in Fig. 2. When the highest frequency, say h, appears in cell C, h is at most the total number of calls from C and its neighboring cells, B and D. The span of frequencies used by the optimal algorithm is at least the maximum among the numbers of calls from B and C and those from C and D. Thus, the upper bound of 2 follows. The lower bound proof is omitted in this paper. □

3 FAL without Deletion

We propose a generic online algorithm Hybrid for FAL in the *without deletion* model. Hybrid consists of two integer parameters, $\alpha \geq 1$ and $\beta \geq 0$. We prove that Hybrid is 3/2-competitive in the absolute case for any $\alpha \geq 1$ and $\beta \geq 0$.

Moreover, with a proper ratio between the values of α and β, the asymptotic competitive ratio of HYBRID is at most 1.382, which is better than Greedy in the asymptotic case.

Conceptually, we divide the frequencies into groups, each of which consists of $\Delta = 3\alpha + \beta$ frequencies. A frequency f is said to be in group i if $\Delta i < f \leq \Delta(i+1)$. HYBRID partitions the set of all frequencies $\{1, 2, \ldots\}$ into 3 disjoint subsets. The first subset F_0 consists of $\alpha + \beta$ frequencies from each group, while each of the remaining 2 subsets F_1 and F_2 has α frequencies. The details of HYBRID are as follows:

Preprocessing Step: The cells of the linear cellular network are partitioned into two sets S_1 and S_2, e.g., cells $C_{2k+1} \in S_1$ and cells $C_{2k} \in S_2$, so that the cells in these two sets are interleaving each other. As mentioned above, the frequencies $\{1, 2, \ldots\}$ are partitioned into 3 disjoint subsets F_0, F_1 and F_2. Precisely, the frequencies of group i for each $i \geq 0$ are distributed to the three subsets as follows.

$$F_0 \leftarrow \{i\Delta + 3j + 1 \mid j = 0, 1, \ldots, \alpha - 1\} \cup \{i\Delta + 3\alpha + j \mid j = 1, \ldots, \beta\}$$
$$F_1 \leftarrow \{i\Delta + 3j + 2 \mid j = 0, 1, \ldots, \alpha - 1\}$$
$$F_2 \leftarrow \{i\Delta + 3j + 3 \mid j = 0, 1, \ldots, \alpha - 1\}$$

Frequency Assignment Step: Suppose a new call emanates from a cell C, which belongs to S_i, we assign a frequency x to the call either from F_i or F_0 according to the following scheme:

$$\min\{x | x \in F_0 \cup F_i, \text{ s.t. } x \text{ is not assigned to cell } C \text{ or any of its neighboring cells}\}$$

3.1 Asymptotic Competitive Ratio

We show that the asymptotic competitive ratio of HYBRID is $(5 - \sqrt{5})/2 \approx 1.382$ when $\alpha/\beta = (\sqrt{5}+1)/2$ and no online algorithm has an asymptotic competitive ratio less than 4/3.

Lemma 1 lower bounds the number of frequencies required by the optimal offline algorithm, i.e., the total number of calls emanating from any two neighboring cells, which helps lead to a bound for the competitive ratio.

Lemma 1. *For a linear cellular network, if a cell A assigns a frequency from group k, then for $\alpha/\beta \geq (1 + \sqrt{5})/2$, the total number of calls from cell A and one of its neighbor is at least $(2\alpha + \beta)k$.*

Theorem 3. *In the without deletion model, the asymptotic competitive ratio of* HYBRID *for FAL approaches $(5 - \sqrt{5})/2 \approx 1.382$ when $\alpha/\beta \to (\sqrt{5} + 1)/2$.*

Proof. If the highest frequency used by HYBRID, say h, is of group k, we have $h \leq (3\alpha+\beta)(k+1)$. Suppose the frequency h is assigned in a cell C. By Lemma 1, C and one of its neighbors together have at least $(2\alpha + \beta)k$ calls when $\alpha/\beta \geq (1 + \sqrt{5})/2$, in which the optimal algorithm has to settle with at least the same amount of frequencies. Therefore, the asymptotic competitive ratio of HYBRID is almost $\lim_{k\to\infty} \frac{(3\alpha+\beta)(k+1)}{(2\alpha+\beta)k} = (5 - \sqrt{5})/2$ when $\alpha/\beta \to (\sqrt{5} + 1)/2$. □

Next, we give a lower bound on the asymptotic competitive ratio for FAL in the without deletion model.

Theorem 4. *No online algorithm for FAL in the without deletion model has an asymptotic competitive ratio less than* 4/3.

Proof. Consider the network in Fig. 2 with cells A, B, C, and D in a row. The adversary initiates n calls from each of cells A and D. For any online algorithm S, S assigns n frequencies to each of A and D. Suppose in each of the two sets of frequencies, xn $(0 \leq x \leq 1)$ of the frequencies do not appear in the other set. Thus, the number of distinct frequencies (span of frequencies used) over the $2n$ frequencies assigned is $(2-x)n$. If $x \leq 2/3$, the adversary stops and we have $R_S^\infty \geq 2 - x \geq 4/3$.

On the other hand, consider the case where $x > 2/3$. The adversary makes n new calls in each of B and C. S must use at least xn new frequencies in each of B and C. By now, S has used at least $(2+x)n$ distinct frequencies. However, the optimal algorithm can satisfy all these calls by $2n$ distinct frequencies. Therefore, $R_S^\infty \geq (2+x)/2 \geq 4/3$. □

3.2 Absolute Competitive Ratio

We show that the absolute competitive ratio of Hybrid is 3/2 for all $\alpha \geq 1$ and $\beta \geq 0$. We also give a matching lower bound proof for the problem, which shows that Hybrid, as well as Greedy, are both optimal.

Theorem 5. *In the without deletion model, the absolute competitive ratio of* Hybrid *algorithm for FAL is at most* 3/2.

Proof. We can prove that Hybrid is 3/2-competitive for all $\alpha \geq 1$ and $\beta \geq 0$. For simplicity, we only prove the competitive ratio for the case $\alpha = 1$ and $\beta = 0$. The general proof will be given in the full paper.

Suppose the highest frequency used by Hybrid, say h, is of group k and assigned by a cell C of S_2 (which is worse than the case of S_1, which uses frequencies from F_1 that has smaller frequency values). We have h either $3k+1$ from F_0 or $3k+3$ from F_2. Consider the former case. C must have assigned k frequencies from F_2 before assigning h. Let $3i+1$ for $i \leq k$ be the highest frequency from F_0 assigned to a neighboring cell of C, say B. B has at least $i+1$ frequencies, where one is from F_0 and i from F_1. On the other hand, C has at least $k-i$ frequencies from F_0. Altogether, B and C consist of at least $k+i+1+k-i$, i.e., $2k+1$, distinct frequencies/calls. The optimal algorithm must use at least the same amount of distinct frequencies. Thus the competitive ratio of Hybrid is at most $(3k+1)/(2k+1) \leq 3/2$.

For the latter case, following the same argument, C has at least $2k-i+1$ distinct frequencies and B, the neighbor of C which has the highest frequency from F_0, has at least $i+1$ distinct frequencies. Then, the optimal algorithm must use at least $2k+2$ distinct frequencies. The competitive ratio of Hybrid is at most $(3k+3)/(2k+2) = 3/2$. □

Next, we give the lower bound of absolute competitive ratio for FAL in the without deletion model.

Theorem 6. *No online algorithm for FAL in the without deletion model has an absolute competitive ratio less than* 3/2.

Proof. The proof is simple. Consider the network in Fig. 2 with cells A, B, C, and D in a row. The adversary begins with one call from each of A and D. For any online algorithm, if it assigns two different frequencies to these two calls, the adversary stops. The competitive ratio of the online algorithm is 2. Otherwise, the same frequency is assigned to both calls. One new call arrives at each of B and C. The online algorithm must use two new frequencies for the two calls. Thus, at least three different frequencies are used, while the optimal algorithm can use only two. Therefore, the absolute competitive ratio is at least 3/2. □

4 FAL with Deletion

In this section we study the online frequency allocation problem in the linear cellular network in which the calls may terminate in arbitrary time. We call this the *with deletion* model. It is noted that the *without deletion* model considered above is a special case of the *with deletion* model. We present a new online algorithm BORROW with competitive ratio at most 5/3. A matching lower bound for problem is given for the absolute case which shows that our algorithm is best possible. For the asymptotic case, we show that no online algorithm has the competitive ratio less than 14/9, which leaves only a small gap between the upper and lower bounds.

4.1 Online Algorithm with Borrowing

The main idea of our algorithm is to reuse ("borrow") an existing frequency even if the frequency is not the smallest possible (i.e., Greedy). Consider Fig. 1. When a call emanates in cell C_i, we try to borrow existing frequencies from C_{i-2} or C_{i+2}, which does not create interference. If none can be borrowed from C_{i-2} or C_{i+2}, the call is satisfied by Greedy. In case there are more than one frequencies that can be borrowed, we select the frequency according to the following priority.

1. The frequency appears in both C_{i-2} and C_{i+2}. If there are more than one of these, pick one arbitrarily.
2. The frequency appears in either C_{i-2} or C_{i+2} which currently has more frequencies that do not appear in C_i. If there are more than one of these, pick one arbitrarily.
3. Pick one arbitrarily.

Theorem 7. *In the with deletion model, the competitive ratio of* BORROW *is at most* 5/3 *for FAL.*

Proof. Consider the network in Fig. 2 with cells A, B, C, D and E in a row. Suppose the highest frequency, say h, is assigned to a call from D. Note that without loss of generality, frequency h is assigned by the greedy approach. Hence, at the time when frequency h is assigned, all frequencies from 1 to h must appear in either C, D or E. We also consider another time instance, which is the latest time before frequency h is assigned, that either C or E assigns a frequency, say h', that does not exist in C or E. Without loss of generality, we assume that it is the cell C to assign the frequency h'. There are only two cases, either C assigns the frequency h' by the greedy approach or frequency h' is borrowed from A. For these two cases, we analyze the competitive ratio of BORROW.

By the greedy approach. Suppose when frequency h is assigned by D, the number of frequencies being used in C, D, E are $y + r_1$, x and $y + r_2$, respectively, where y is the number of common frequencies among cells C and E. Since frequency h is assigned by the greedy approach, we have $h = x + y + r_1 + r_2$, which is the number of distinct frequencies used in the three cells. In fact, for any algorithm to satisfies all calls from these cells, one has to use at least $x + y + \max\{r_1, r_2\}$ distinct frequencies.

Suppose when frequency h' is assigned by C, the number of frequencies being used in C and E are $y' + r'_1$ and $y' + r'_2$, respectively, where y' is the number of common frequencies among C and E. Note that as there are r'_2 frequencies in E that C did not borrow, the r'_2 frequencies must be used in B. Hence, the number of frequencies, and thus the number of calls, from cells B and C is at least $y + r_1 + r_2$. By the definition of frequency h', at the time frequency h' is assigned, the number of distinct frequencies among C and E, i.e., $y' + r'_1 + r'_2$, must be at least $y + r_1 + r_2$. Any algorithm to satisfy the calls from B and C has to use at least $y' + r'_1 + r'_2 \geq y + r_1 + r_2$ frequencies.

As a result the competitive ratio of BORROW is at most

$$\frac{x + y + r_1 + r_2}{\max\{x + y + \max\{r_1, r_2\}, y + r_1 + r_2\}} \leq \frac{3}{2}.$$

By borrowing. Similar to the previous case, suppose when frequency h is assigned by D, the number of frequencies being used in C, D, E are $y + r_1$, x and $y + r_2$, respectively, where y is the number of common frequencies among C and E. For any algorithm to satisfies all calls from these cells, one has to use at least $x + y + \max\{r_1, r_2\}$ distinct frequencies. Suppose when frequency h' is assigned by C, the number of frequencies being used in C and E are $y' + r'_1$ and $y' + r'_2$, respectively, where y' is the number of common frequencies among C and E.

In this case, frequency h' assigned by C is borrowed from A but not E. There are two subcases by the algorithm: either all the r'_2 frequencies in E which could be assigned to C are already in B (i.e., E has no candidate for C to borrow) or the number of frequencies in A is at least that in E. For the former subcase, we have the number of frequencies in B and C at least $y' + r'_1 + r'_2$, and hence the analysis follows the previous case which yields a competitive ratio at most $3/2$. For the latter subcase, we have the number of frequencies in A but not in C at least that of frequencies in E but not in C, which is r'_2. In addition, what those frequencies that A could have but not in C are one frequency that is borrowed to C, and also the frequencies that are neither in C nor E and with frequency value less than h. There are at most x of them. That implies that $r'_2 \leq x$. Therefore, the competitive ratio of our algorithm is at most

$$\frac{x + y + r_1 + r_2}{\max\{x + y + \max\{r_1, r_2\}, y' + r'_1\}}$$

with the constraints that $r'_2 \leq x$ and $y + r_1 + r_2 \leq y' + r'_1 + r'_2$. By the constraints, we have $y' + r'_1 \geq y + r_1 + r_2 - x$. Together with the fact that $\max\{r_1, r_2\} \geq (r_1 + r_2)/2$, we can prove that the ratio is at most 5/3. □

4.2 Lower Bound

Theorem 8. *There is no online algorithm for FAL, in the with deletion model, with an absolute competitive ratio less than* 5/3 *or an asymptotic competitive ratio less than* 14/9.

Proof. For the absolute competitive ratio, we give an adversary that any online algorithm will use at least five distinct frequency (with span of at least five), while the optimal algorithm uses only three.

Consider the network Fig. 2 with cells A, B, C, D, E and F in a row. The adversary has three calls emanate from each of A, C and F. In order for an algorithm to use less than five distinct frequency, either the sets of frequency in A and C differ by one frequency or the two sets are the same. In the following, we analyze these two cases to show that no online algorithm has an absolute competitive ratio less than 5/3.

- If the sets of frequencies in A and C differ by one frequency, without loss of generality, we can assume that the set of frequencies in A is $\{1, 2, 3\}$ and that in C is $\{1, 2, 4\}$. In that case, the adversary terminates frequency 1 in A and frequency 2 in C, and make a call from B in which the fifth distinct frequency, say 5, has to be used. It is easy to see that the optimal can make use of three distinct frequencies only, and hence the competitive ratio is at least 5/3.
- If the sets of frequencies in A and C is the same, without loss of generality, we can assume that both sets of frequency are $\{1, 2, 3\}$. Moreover, if less than five distinct frequencies are used, the sets of frequency in F must be in the form $\{1, 2, 3, 4\} - \{i\}$ for a fixed i with $1 \leq i \leq 4$. The aim of the adversary is to make a call in B such that frequency i must be assigned to serve the call. This can be done by terminating all calls in A except one and all calls in C except one, such that the remaining calls in A and C use a different frequency and none of the two frequencies are frequency i. Note that this can always be done since originally there are three calls in each of A and C. After frequency i is assigned by B, all calls in A and C are terminated and two new calls are made from B and three new calls are made from D. Since frequency i is used in B but not in F, the three frequencies assigned by D

cannot be the same to both of those in B and F. Then, applying the same argument as in the previous case, we can show that the online algorithm must use at least five distinct frequencies while the optimal algorithm can use only three. Hence, the competitive ratio is at least 5/3.

For the asymptotic competitive ratio, the adversary makes n calls from each of the A, C and F. Let f_X denote the set of frequencies in cell X. For any online algorithm, let γ be the minimum between the numbers of common frequencies in A and F, and C and F, i.e., $\gamma = \min\{|f_A \cap f_F|, |f_C \cap f_F|\}$. The online algorithm uses at least $2n - \gamma$ distinct frequencies.

The adversary then terminates some calls in A and C such that $f_A \cap f_C = \emptyset$ and $f_A \cup f_C \subseteq f_F$ and $|f_A| = |f_C| = \gamma/2$. After that, $n - \gamma/2$ new calls are made from B, in which at least $\gamma/2$ of the frequencies assigned will not be in F. Then, all calls from A and C are terminated, $\gamma/2$ and n new calls are made from B and D, respectively. Since at least $\gamma/2$ of the frequencies in B are not in F and vice versa, D has at least $\gamma/4$ frequencies either not in B or F and vice versa, and without loss of generality assume that it is F. The adversary terminates some calls in D and F such that $f_D \cap f_F = \emptyset$ and $|f_D| = |f_F| = n/2 + \gamma/8$. Then, $n/2 - \gamma/8$ new calls are made from E in which the frequencies assigned must be different from those currently in D and F. The online algorithm must use at least $3n/2 + \gamma/8$ distinct different to satisfy all the calls in D, E and F. Including the case where γ is defined, the online algorithm uses at least $\max\{2n - \gamma, 3n/2 + \gamma/8\} \geq 14n/9$ distinct frequencies. As the optimal algorithm can use only n frequencies to satisfy all calls, the competitive ratio of the online algorithm is at least 14/9. □

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