Socially-optimal Multi-hop Secondary Communication under Arbitrary Primary User Mechanisms

Hongxing Li*, Chuan Wu*, Zongpeng Li† and Francis C.M. Lau*

*Department of Computer Science, The University of Hong Kong, Hong Kong, {hxli, cwu, fcmlau}@cs.hku.hk

†Department of Computer Science, University of Calgary, Canada, zongpeng@ucalgary.ca

Abstract—In a cognitive radio system, licensed primary users can lease idle spectrum to secondary users for monetary remuneration. Secondary users acquire available spectrum for their data delivery needs, with the goal of achieving high throughput and low spectrum charges. Maximizing such a net utility (throughput utility minus spectrum cost) is a central problem faced by a multi-hop secondary network. Optimal decision making is challenging, since it involves multiple data flows, cross-layer coordination, and economic constraints (budgets of sources). The picture is further complicated by the inter-play between secondary data communication and primary spectrum leasing mechanisms. This work is the first to investigate the full spectrum of socially optimal secondary user communication. We design a social welfare maximization framework for multi-session multi-hop secondary data dissemination based on Lyapunov optimization techniques. A salient feature of the framework is that it takes any given primary user mechanism as input, and produces correspondingly a dynamic, distributed rate control, routing, and spectrum allocation and pricing protocol that can achieve long-term maximization of the overall system utility. Through rigorous theoretical analysis, we prove that our online protocol can achieve a social welfare that is arbitrarily close to the offline optimum, with only finite buffer space requirement at each secondary user, and guarantee of no buffer overflow. Empirical studies are conducted to examine the performance of the protocol.

I. INTRODUCTION

Cognitive radio techniques have emerged as a promising approach to more effectively exploit the under-utilized wireless spectrum, and hence to mitigate the spectrum scarcity problem. Licensed primary users can periodically pool fallow channel spectrum for sale to unlicensed secondary users [2], [3], [10]. Monetary payments from secondary users can serve as an important motivation for primary users to relinquish their spectrum. A number of market mechanisms, usually in the form of an auction, have been designed to assist such spectrum transactions [6], [8], [11], [22], [23], [24]. Secondary users purchase spectrums for their data delivery needs, with a natural goal of minimizing spectrum charges while maximizing end-to-end throughput utility.

When a secondary user network hosts multiple multi-hop communication sessions, a fundamental problem arises: how should the secondary users share the available spectrums, jointly route the traffic from different sessions, and adjust the end-to-end rates of the sessions, in order to maximize the overall net utility (end-to-end throughput utility minus spectrum cost)? The problem is challenging, given that we aim at a dynamic algorithm with social welfare maximization guarantee over a long run of the system and under volatile spectrum occupancy patterns of the primary users as well as dynamic network connectivity and capacity.

The challenge is further aggravated when we target a flexible algorithm that can work with any spectrum selling mechanism at the primary users, and is always able to produce maximum social welfare for the entire secondary user community. Current literature [5], [14], [20], [21] on throughput/utility maximizing cross-layer design in secondary user networks mostly assume free spectrum-sharing through opportunistic sensing, but without monetary payments. These studies are not intended for any specific spectrum selling mechanism at the primary users, let alone the applicability to general spectrum selling mechanisms. On the other hand, there have been studies on primary user mechanism design, *e.g.*, spectrum auction mechanisms [6], [8], [11], [22], [23], which usually assume static routes and data rates among secondary users, and hence avoid the routing and rate control problems.

The picture becomes even more complicated, but at the same time more practical if we also consider the budget constraints at the secondary users. Existing work on spectrum auctions in cognitive radio networks commonly assume unlimited budgets at the participating secondary users [6], [8], [11], [22], [23], [24]. In this work, each secondary sender is practically furnished with a limited average budget, to pay for spectrums along its data delivery paths towards the destination. A new dimension of the challenge thus arises on how to dynamically manage the spectrum sharing and pricing such that the average costs do not exceed the average budgets, while social welfare maximization is still guaranteed.

In this work, we design a social welfare optimization framework for multiple-unicast data communication in a multi-hop secondary user network, which explores the full spectrum of secondary user communication under any given primary user mechanism. The framework produces a dynamic, joint rate control, routing, and spectrum sharing and pricing protocol for the secondary users to execute in either a centralized or a distributed fashion. Given any spectrum selling mechanism at the primary users (*e.g.*, a spectrum auction), which dictates the costs of the spectrums, the online protocol guarantees overall net utility maximization over a long running span of the system. Rooted in Lyapunov optimization theory [16], such social welfare maximization and the stability of the network are achieved by dynamic scheduling of transmissions among packet queues at the secondary users. The contributions of this work are as follows:

1 We propose a social welfare maximization framework for multi-hop multi-session unicast communication in secondary user networks with practical budget constraints, and design a dynamic, cross-layer optimization protocol that caters to any

spectrum selling mechanism at the primary users.[1]

2 We rigorously prove that the social welfare achieved by our dynamic algorithm can be arbitrarily close to the offline optimum which is derived with complete knowledge of the system over its long course of running. Our algorithm permits flexible tradeoffs between (i) optimality and (ii) the required buffer spaces and the allowed worst-case budget deficits at the secondary users (*i.e.*, the amount of budget a node borrows from its future income for spectrum purchases). The optimality can still be guaranteed when dynamic decisions at each relay node along the unicast paths are made using delayed information of budget deficits from the source nodes. Empirical studies are conducted under practical settings to further examine the algorithm performance and the tradeoffs of social welfare optimality versus buffer usages and worst-case budget deficits.

3 For the first time in the literature of Lyapunov optimization, our dynamic protocol ensures not only finite buffer sizes for all packet queues, with no-buffer-overflow guarantee, but also finite sizes of virtual queues, *e.g.*, finite budget deficits, for all unicast sessions in the worst cases.

The rest of the paper is organized as follows. We discuss related work in Sec. II, introduce the optimization framework in Sec. III, give details of the dynamic protocol design in Sec. IV, evaluate the protocol performance with theoretical analysis and empirical studies in Sec. V and Sec. VI, respectively, and finally conclude the paper in Sec. VII.

II. RELATED WORK

Spectrum auction [4] is the most commonly adopted spectrum selling mechanism by primary users in cognitive radio networks, because of its fairness and efficiency in spectrum allocation and pricing. A rich body of work exists on designing spectrum auction mechanisms with one or multiple of the following objectives ([6], [8], [11], [22], [23], [24] and references therein): truthfulness, fairness, high revenue for primary users, high spectrum utilization, and (approximate) optimal social welfare. Besides auctions, other market mechanisms, *e.g.*, game-based [18] and contract-based [7] ones, are also explored for efficient spectrum selling at the primary users. Most spectrum selling mechanisms assume unlimited budgets, given data rates and fixed routes of the secondary users. A recent work of Zhu *et al.* [24] jointly considers spectrum auction and routing in multi-hop secondary networks, which however is only applicable to a specific truthful auction for spectrum selling without budget concerns.

Shi *et al.* [20] propose a distributed throughput optimization algorithm, which iteratively increases date rates for user communication sessions in multi-hop cognitive radio networks. Feng *et al.* [5] introduce a two-phase distributed protocol with primal-dual decomposition. Xue *et al.* [21] propose a throughput maximization protocol, under the constraints of bounded collision rates between secondary and primary users.

Li *et al.* [14] present a cross-layer algorithm maximizing the throughput utility in a socially selfish secondary network. These work focus on opportunistic spectrum access without monetary payments to the primary users, and do not adapt to the spectrum selling mechanisms at the primary users.

Classic protocol designs based on Lyapunov theory often employ buffers with infinite sizes [16]. The challenge of using finite buffers is only being considered recently. Le *et al.* [13] investigate the optimal control of a wireless network with a finite buffer for each by-passing session per relay node; but an infinite buffer is still necessary at each source node in the worst cases. Neely [17] presents an opportunistic scheduling protocol with a bounded-size buffer at each node for each data session, by simply dropping the packets when a buffer is full. We propose in our previous work [14] a utility-maximizing algorithm with finite packet queues but infinite virtual queues. This work advances the state-of-the-art by providing bounded sizes for not only each packet queue on each node, but also for all virtual queues (*e.g.*, budget deficits), while guaranteeing close-to-optimum social welfare and no buffer-overflow.

III. SYSTEM MODEL AND PRELIMINARIES

We first describe our network model (III-A), the layers of the network stack under investigation (III-B), and a generic primary user mechanism (III-C). The budget model at secondary users will also be explained (III-D).

A. Network Model

Consider a cognitive radio system $G = (V_P, V_S, \mathcal{E})$ with a set of primary users V_P and a set of secondary users V_S, distributed in a given geographical area. The secondary users constitute a multi-hop *secondary user network*. A directed edge $e_{ij} \in \mathcal{E}$ implies that node $i \in V_S$ can transmit directly to node $j \in V_S$, over a given channel. Each primary user $v \in V_P$ has a distinct licensed channel. The set of channels is $\mathcal{C}$, and $|\mathcal{C}| = |V_P|$.

A set of unicast sessions $\mathcal{M}$ is defined over the secondary user network. A session m is from source $s_m \in V_S$ to destination $d_m \in V_S$, and in general requires multi-hop routing and relaying, assisted by other secondary users.

We consider a generic interference model. Let I denote the set of interference relations among potential transmissions in the system. It includes two types of pairs: (1) $(e_{ij}, e_{kl}) \in I$ implies that the two transmissions e_{ij} and e_{kl} can not occur concurrently on the same channel; (2) $(v_p, e_{ij}) \in I$ (with $v_p \in V_P$ and $e_{ij} \in \mathcal{E}$) implies that when a primary user v_p is actively using its licensed channel, transmission e_{ij} cannot simultaneously happen at the same channel. We also assume that each secondary user is equipped with a single radio, such that it may either transmit or receive data on one channel at each time. Such a generic model I subsumes most interference models in the literature, including the node-exclusive model and the k-hop ($k \geq 1$) interference model [19].

Important notations are summarized in Table I.

---

[1]To the best of our knowledge, this is the first work investigating the impact of spectrum selling mechanisms on protocol design for throughput/utility maximization in secondary networks.

V_P	Set of primary users	V_S	Set of secondary users
$\mathcal{E}$	Set of links	G	(Vp, Vs, ϵ)
$\mathcal{C}$	Set, orthogonal channels	$\mathcal{M}$	Set of unicast sessions
e_{ij}	Directed $i \rightarrow j$ link	I	Set, interference relations
$\mathbb{E}(\cdot)$	The expectation	$U(\cdot)$	Utility function
s_m	Source of session m	d_m	Destination of session m
C_i	Available channel set at secondary node i		
$z_{ij}^{(c)}(t)$	Price to relay unit data on channel c over e_{ij} in time slot t		
z_{max}	Maximum price for spectrum leasing		
z_{min}	Minimum price for spectrum leasing		
$A_m(t)$	Data arrival rate of session m in time slot t		
$A_{max}^{(m)}$	Maximum data arrival rate of session m		
$b_m(t)$	Budget arrival rate of session m in time slot t		
$b_{max}^{(m)}$	Maximum budget arrival rate of session m		
$r_m(t)$	Admissible data rate of session m in time slot t		
$\eta_m(t)$	Auxiliary variable of session m in time slot t		
$\mu_{ij}^{(m)}(t)$	Binary var: data session m is routed over e_{ij} in time slot t?		
$\alpha_{ij}^{(c)}(t)$	Binary var: channel c is assigned to e_{ij} in time slot t?		
$F_{ij}^{(c)}(\cdot)$	Primary user mechanism: spectrum allocation for $\alpha_{ij}^{(c)}(t)$		
$S_{ij}^{(c)}$	Set of values for price $z_{ij}^{(c)}(t)$ leading to $\alpha_{ij}^{(c)}(t)$		
$F_{ij}^{(c)^{-1}}(\cdot)$	Primary user mechanism: spectrum prices for $S_{ij}^{(c)}$		
$Q_i^{(m)}(t)$	Data queue of session m on user i in time slot t		
$q_i^{(m)}$	Buffer size for data queue $Q_i^{(m)}$ of session m on user i		
$H_m(t)$	Budget deficit of session m in time slot t		
$Y_m(t)$	Transport virtual queue of session m at time t		
V	User-defined positive constant in Lyapunov Optimization		
B	Quantity defined in Lyapunov Optimization in Sec. IV		

TABLE I
LIST OF NOTATIONS.

B. The Three Protocol Layers

1) Transport layer at the sources: At the source of each unicast session, end-to-end rate control is considered at the transport layer. Suppose the system runs in a time-slotted fashion [13], [14], [21]. In each time slot t, a random number $A_m(t) \in [0, A_{max}^{(m)}]$ of data units are generated at the application layer of source s_m, to be admitted to the transport layer (we ignore header overhead). For *rate control*, let $r_m(t) \in [0, A_m(t)]$ be the amount of data admitted to the network in a time slot, such that congestion will not occur and *network stability* (formally defined in III-E) is achieved.

2) Network layer at each secondary user: Each secondary user $i \in V_S$ may receive data from multiple sessions (including one originating from itself), and makes routing decisions to forward them toward respective destinations. Each relay node maintains a packet queue $Q_i^{(m)}$, which is a network-layer buffer, for each session $m \in \mathcal{M}$ where i is not the destination d_m of the session. Destination d_m directly delivers data of session m to its upper layers without buffering. The queueing law for each $Q_i^{(m)}$ is:

$$Q_i^{(m)}(t+1) = \max\{Q_i^{(m)}(t) - \sum_{e_{ij} \in \mathcal{E}} \mu_{ij}^{(m)}(t), 0\} + \sum_{e_{ji} \in \mathcal{E}} \mu_{ji}^{(m)}(t)$$
$$+ \mathbf{1}_{\{i=s_m\}} r_m(t), \ \forall m \in \mathcal{M}, i \in V_S, i \neq d_m. \quad (1)$$

Here, $\mu_{ij}^{(m)}(t) \in \{0, 1\}$ is the amount of data routed over link $e_{ij} \in \mathcal{E}$ for session $m \in \mathcal{M}$ in time slot t. We assume that all transmission links are of equal capacity, and the length of a time slot is just sufficient for a link to transmit one unit of data [13], [14], [21]. $\mathbf{1}_{\{n=s_m\}}$ is a binary function, where $\mathbf{1}_{\{n=s_m\}} = 1$ if $n = s_m$ and 0 otherwise. Let non-negative constant $q_i^{(m)}$ be the buffer size for queue $Q_i^{(m)}(t)$. We will

show that our algorithm proposed in Sec. IV guarantees finite buffer sizes without overflow.

3) MAC layer at each secondary user: Based on routing decisions from the network layer, a channel allocation and link scheduling scheme operates at the MAC layer, to schedule transmissions on each available channel in each time slot.

Let $\alpha_{ij}^{(c)}(t) \in \{0, 1\}$ indicate whether link $e_{ij} \in \mathcal{E}$ is scheduled on channel $c \in \mathcal{C}$ in t, with 0 for 'no' and 1 for 'yes'. The following constraints apply in each time slot:

$$\sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) = \sum_{m \in \mathcal{M}} \mu_{ij}^{(m)}(t), \forall e_{ij} \in \mathcal{E}, \quad (2)$$

$$\sum_{c \in \mathcal{C}} [\sum_{e_{ij} \in \mathcal{E}} \alpha_{ij}^{(c)}(t) + \sum_{e_{ji} \in \mathcal{E}} \alpha_{ji}^{(c)}(t)] \leq 1, \forall i \in V_S, \quad (3)$$

$$\alpha_{ij}^{(c)}(t) + \alpha_{kl}^{(c)}(t) \leq 1, \forall e_{ij}, e_{kl} \in \mathcal{E}, (e_{kl}, e_{ij}) \in I, c \in \mathcal{C}, \quad (4)$$

$$\alpha_{ij}^{(c)}(t) \leq \mathbf{1}_{ij}^{(c)}(t), \forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}, \quad (5)$$

$$\alpha_{ij}^{(c)}(t) \in \{0, 1\}, \forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}, \quad (6)$$

Here $\mathbf{1}_{ij}^{(c)}(t)$ is a binary function, where $\mathbf{1}_{ij}^{(c)}(t) = 1$ if channel c is available to e_{ij} in time slot t, and 0 otherwise. Constraint (2) states that the total amount of data transmitted from i to j on different channels, in one time slot, should be equal to the total units of data from all sessions to be routed from i to j in that time slot. Inequality (3) models the primary interference constraint: a node can either transmit or receive on at most one channel in each time slot. Constraints (4) and (5) model the interference relations in I: the former indicates that interfering links can not be concurrently active over the same channel; the latter states that a link transmission over a given channel is possible only if that channel is available. A channel c is available to link e_{ij} in time slot t if the primary user v_p owning c is not transmitting in t, or no interference exists between the transmissions from v_p and along e_{ij}, *i.e.*, $(v_p, e_{ij}) \notin I$.

C. A Generic Primary User Mechanism

Let $z_{ij}^{(c)}(t)$ denote the price paid to a primary user for using channel $c \in \mathcal{C}$ to transmit one unit of data over link $e_{ij} \in \mathcal{E}$ in time slot t, by the source of a unicast session. The spectrum selling decisions at the primary users are in general related to the prices and the current network status, including network topology G and interference constraints I. Once the prices are determined, the primary users can decide a collision-free channel allocation for the secondary data communication.

In order to derive a generic framework for social welfare maximization, we do not assume a specific primary user mechanism, but employ a set of functions $F_{ij}^{(c)}(G, I, \mathcal{C}, \mathbf{Z}(t)), \forall e_{ij} \in \mathcal{E}, \forall c \in \mathcal{C}$ to characterize the channel allocation decisions of the primary users. Here, the network status $(G, I, \mathcal{C})$ and the set of prices $\mathbf{Z}(t) = \{z_{ij}^{(c)}(t) | e_{ij} \in \mathcal{E}, c \in \mathcal{C}\}$ are inputs, and channel allocation decisions are made as $\alpha_{ij}^{(c)}(t) = F_{ij}^{(c)}(G, I, \mathcal{C}, \mathbf{Z}(t)), \forall e_{ij} \in \mathcal{E}, \forall c \in \mathcal{C}$. We assume that the spectrum selling mechanism, *i.e.*, functions $F_{ij}^{(c)}(G, I, \mathcal{C}, \mathbf{Z}(t))$, are known to the secondary users. We use $F_{ij}^{(c)^{-1}}(G, I, \mathcal{C}, \boldsymbol{\alpha}(t))$ to denote a vector-valued inverse

function of $F_{ij}^{(c)}(\cdot)$, which outputs a set of possible values for price $z_{ij}^{(c)}(t)$, and setting $z_{ij}^{(c)}(t)$ to any value in ${F_{ij}^{(c)}}^{-1}(G, I, \mathcal{C}, \boldsymbol{\alpha}(t)), \forall e_{ij} \in \mathcal{E}, \forall c \in \mathcal{C}$, can lead to channel allocation decisions $\boldsymbol{\alpha}(t) = \{\alpha_{ij}^{(c)}(t) | e_{ij} \in \mathcal{E}, c \in \mathcal{C}\}$, given network status $(G, I, \mathcal{C})$.

Let z_{min} and z_{max} be the minimum and maximum spectrum leasing prices, respectively, of a channel successfully leased in one time slot. A rational spectrum selling mechanism exhibits the following two properties:

(i) When a price $z_{ij}^{(c)}(t)$ is set to 0, channel c is not leased to any transmission over link e_{ij} in time slot t, since a rational primary user has no motivation to lease its channel for free, i.e., $\forall e_{ij} \in \mathcal{E}, \forall c \in \mathcal{C}$,

$$F_{ij}^{(c)}(G, I, \mathcal{C}, \mathbf{Z}(t)) = 0, \text{ if } z_{ij}^{(c)}(t) = 0. \tag{7}$$

(ii) No charge is incurred on link e_{ij} for leasing channel c in t, unless c is allocated to e_{ij} in t ($\alpha_{ij}^{(c)}(t) = 0$) by the corresponding primary user, i.e., $\forall e_{ij} \in \mathcal{E}, \forall c \in \mathcal{C}$,

$$ {F_{ij}^{(c)}}^{-1}(G, I, \mathcal{C}, \boldsymbol{\alpha}(t)) = \{0\}, \text{ if } \alpha_{ij}^{(c)}(t) = 0. \tag{8}$$

D. Budget Model

In each time slot, a random amount of budget $b_m(t) \in [0, b_{max}^{(m)}]$ is provided to a source $s_m, \forall m \in \mathcal{M}$, to pay for the spectrum lease (in t and/or later time slots) for transmissions along all paths from s_m to d_m. Here, $b_{max}^{(m)}$ is the maximum budget provision rate for session m. A practical implication of this random budget arrival is that a real-world secondary user may allocate different budgets for data transmission at different times. We seek to maximize the social welfare of all unicast sessions while guaranteeing that each session's time-averaged expenditure on spectrum leasing is no larger than the time-averaged budget provision at its source, i.e.,

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\Big(\sum_{e_{ij} \in \mathcal{E}} \big(\mu_{ij}^{(m)}(t) \cdot \sum_{c \in \mathcal{C}} (\alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t)) \big) \Big)$$
$$\leq \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}(b_m(t)), \qquad \forall m \in \mathcal{M}. \tag{9}$$

Here $\sum_{e_{ij} \in \mathcal{E}} (\mu_{ij}^{(m)}(t) \cdot \sum_{c \in \mathcal{C}} (\alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t)))$ is the overall cost on spectrum leasing of data session m during t.

E. Some Definitions and Preliminaries

We conclude this section by introducing some useful definitions and existing results.

Definition 1: The time averaged value of a time-varying variable $x(t)$ is denoted by

$$\bar{x} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}(x(\tau)).$$

Definition 2 (Queue and Network Stability): A queue Q is *strongly stable* (or *stable* for short) if and only if $\lim_{t \to \infty} \sup \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}(Q(\tau)) < \infty$, where $Q(\tau)$ is the queue size in time slot τ. A network is *strongly stable* (or *stable* for short) if and only if all queues in the network are stable.

Theorem 1 (Necessity for Queue Stability): For any queue Q with the following queuing law,

$$Q(t + 1) = \max\{Q(t) - b(t), 0\} + a(t),$$

where $a(t)$ and $b(t)$ are the queue incoming rate and outgoing rate in time slot t, respectively, the following result holds:

If queue Q is strongly stable, then its average incoming rate $\bar{a} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}(a(\tau))$ is no larger than the average outgoing rate $\bar{b} = \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}(b(\tau))$.

IV. THE DYNAMIC ALGORITHM

We next introduce the social welfare maximization problem (IV-A). We design a dynamic cross-layer control algorithm to solve the problem (IV-B,IV-C), and discuss its distributed implementation (IV-D).

A. Social Welfare Maximization

Our objective is to maximize the overall time-averaged social welfare of all unicast sessions in the secondary user network, under given budget constraints and any given spectrum selling mechanism at the primary users, while guaranteeing network stability. The social welfare is defined as the summation of the time-averaged throughput utility of all unicast sessions subtracting the overall time-averaged cost for spectrum leasing:

$$\phi = \sum_{m \in \mathcal{M}} U(\bar{r}_m) - \sum_{c \in \mathcal{C}} \sum_{e_{ij} \in \mathcal{E}} \overline{\alpha_{ij}^{(c)} \cdot z_{ij}^{(c)}}.$$

Here $\bar{r}_m$ is the time-averaged end-to-end data rate of session m; $U(\cdot)$ is a non-decreasing, differentiable and concave utility function on $\bar{r}_m$; $\overline{\alpha_{ij}^{(c)} \cdot z_{ij}^{(c)}}$ is the time-averaged expense of using channel c at link e_{ij}, among all the sessions.

The social welfare maximization problem then is:

$$\max_{r_m, \mu_{ij}^{(m)}, \alpha_{ij}^{(c)}, z_{ij}^{(c)}} \phi \tag{10}$$

$$s.t. \qquad \mathbf{r} \in \Lambda, \tag{11}$$

$$\text{budget constraints in (9).}$$

Λ is the capacity region and $\mathbf{r} = (\bar{r}_1, \ldots, \bar{r}_M) \in \Lambda$ means that there exists a set of feasible routing, channel allocation and pricing strategies, $\mu_{ij}^{(m)}(t)$, $\alpha_{ij}^{(c)}(t)$ and $z_{ij}^{(c)}(t)$, $\forall e_{ij} \in \mathcal{E}, \forall m \in \mathcal{M}, \forall c \in \mathcal{C}$, satisfying constraints (2)-(8), which decide a set of feasible admissible data rates in $\mathbf{r}$, such that all queues in the network are stable.

B. Modeling Virtual Queues

We apply Lyapunov optimization techniques to design a dynamic algorithm to solve the social welfare maximization problem, which decides $r_m(t)$, $\mu_{ij}^{(m)}(t)$, $\alpha_{ij}^{(c)}(t)$ and $z_{ij}^{(c)}(t)$, $\forall e_{ij} \in \mathcal{E}, m \in \mathcal{M}, c \in \mathcal{C}$, in each time slot t, and guarantees that time averages of these quantities maximize the social welfare. We first introduce two types of virtual queues, for end-to-end rate control and for the budget constraints, respectively.

Virtual queue for rate control. In the Lyapunov optimization framework [16], it is difficult to directly maximize a non-linear function $U(\bar{r}_m)$ over a time-averaged quantity $\bar{r}_m$, but a technique of introducing virtual queues can be applied to establish a lower bound of $U(\bar{r}_m)$: a virtual queue $Y_m(t)$ and an auxiliary variable $\eta_m(t)$ are modeled and maintained at source s_m, $\forall m \in \mathcal{M}$, as follows

$$Y_m(t + 1) = \max\{Y_m(t) - r_m(t), 0\} + \eta_m(t), \tag{12}$$

under constraints

$$0 \leq \eta_m(t) \leq A_{max}^{(m)}, \ 0 \leq r_m(t) \leq A_m(t). \quad (13)$$

The rationale is that, our algorithm seeks to maintain the stability of $Y_m(t)$, and then $\bar{\eta}_m \leq \bar{r}_m$ is guaranteed according to Theorem 1. Hence $U(\bar{\eta}_m)$ is a lower bound of $U(\bar{r}_m)$, and we can maximize $U(\bar{\eta}_m)$ in order to approximately maximize $U(\bar{r}_m)$ in (10).

Virtual queue for satisfying budget constraint. To ensure the budget constraints in (9) by controlling decision variables in each time slot, we introduce another virtual queue $H_m(t)$ at each source s_m, with queueing law:

$$H_m(t+1) = \max\{H_m(t) - b_m(t), 0\} + \sum_{e_{ij} \in \mathcal{E}} (\mu_{ij}^{(m)}(t) \cdot \sum_{c \in \mathcal{C}} (\alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t))), \ \forall m \in \mathcal{M}. \quad (14)$$

The queue backlog $H_m(t+1)$ represents the cumulative *budget deficit* of session m at the beginning of time slot $t+1$, *i.e.*, the total expense incurred by spectrum leasing of session m in t ($\sum_{e_{ij} \in \mathcal{E}} \mu_{ij}^{(m)}(t) \cdot \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t)$) minus the amount of injected budget at the session's source in t ($b_m(t)$), plus any budget deficit carried forward from t ($H_m(t)$).

If this queue is stable, we can claim that the time-averaged spectrum charge is no larger than the time-average budget provision in each session, *i.e.*, constraints in (9) are satisfied, according to Theorem 1.

C. Dynamic Algorithm Design

In our dynamic algorithm, three types of queues $\Theta(t) = \{\mathbf{Q}(t), \mathbf{H}(t), \mathbf{Y}(t)\}$ are maintained, with $\mathbf{Q}(t) = \{Q_i^{(m)}, \forall m \in \mathcal{M}, i \in \mathcal{V}_S, i \neq d_m\}$, $\mathbf{H}(t) = \{H_m(t), \forall m \in \mathcal{M}\}$ and $\mathbf{Y}(t) = \{Y_m(t), \forall m \in \mathcal{M}\}$. Define the Lyapunov function [16] as

$$L(t) = \frac{1}{2} \Big[\sum_{m \in \mathcal{M}} \sum_{i \in V_S, i \neq d_m} \frac{q_{s_m}^{(m)} \cdot (Q_i^{(m)}(t))^2}{q_i^{(m)}} + \sum_{m \in \mathcal{M}} (H_m(t))^2 + \sum_{m \in \mathcal{M}} (Y_m(t))^2 \Big].$$

The one-slot conditional Lyapunov drift is defined as:

$$\Delta(t) = L(t+1) - L(t).$$

Squaring Eqn. (1), (12) and (14), we derive the following inequality (detailed steps in our technical report [15]):

$$\Delta(t) - V \cdot \Big(\sum_{m \in \mathcal{M}} U(\eta_m(t)) - \sum_{c \in \mathcal{C}} \sum_{e_{ij} \in \mathcal{E}} \alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t) \Big)$$

$$\leq B - \sum_{m \in \mathcal{M}} b_m(t) \cdot H_m(t) - \Psi_1(t) - \Psi_2(t) - \Psi_3(t), \quad (15)$$

where V is a user-defined positive constant that can be interpreted as the weight of utility, and $B = \frac{1}{2} \sum_{m \in \mathcal{M}} [\sum_{i \in V_S, i \neq d_m} \frac{q_{s_m}^{(m)}}{q_i^{(m)}} [(\mathbf{1}_{\{i=s_m\}} A_{max}^{(m)} + 1)^2 + 1] + (b_{max}^{(m)})^2 + (\frac{|V_S|}{2} z_{max})^2 + 2(A_{max}^{(m)})^2]$ is a constant value. $\Psi_1(t)$, $\Psi_2(t)$, $\Psi_3(t)$ are as follows:

$$\Psi_1(t) = \sum_{m \in \mathcal{M}} (V \cdot U(\eta_m(t)) - \eta_m(t) \cdot Y_m(t)),$$

which is only related to auxiliary variables $\eta_m(t), \forall m \in \mathcal{M}$;

$$\Psi_2(t) = \sum_{m \in \mathcal{M}} r_m(t) \cdot (Y_m(t) - Q_{s_m}^{(m)}(t)),$$

which is only related to rate control variables $r_m(t), \forall m \in \mathcal{M}$;

$$\Psi_3(t) = \sum_{e_{ij} \in \mathcal{E}} \Big[\sum_{m \in \mathcal{M}} \mu_{ij}^{(m)}(t) \cdot W_{ij}^{(m)}(t) - V \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t) \Big],$$

with

$$W_{ij}^{(m)}(t) = q_{s_m}^{(m)} \cdot \Big[\frac{Q_i^{(m)}(t)}{q_i^{(m)}} - \frac{Q_j^{(m)}(t)}{q_j^{(m)}} \Big] - H_m(t) \cdot \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t), \quad (16)$$

which is related to the routing, channel allocation and pricing variables $\mu_{ij}^{(m)}(t)$, $\alpha_{ij}^{(c)}(t)$ and $z_{ij}^{(c)}(t), \forall e_{ij} \in \mathcal{E}, m \in \mathcal{M}, c \in \mathcal{C}$.

Based on the *drift-plus-penalty* framework [16], we derive the following dynamic algorithm that observes queues $\Theta(t) = \{\mathbf{Q}(t), \mathbf{H}(t), \mathbf{Y}(t)\}$ at each time slot t and makes control decisions that minimize the RHS of (15), such that a lower bound for the social welfare in (10) is maximized. Since $B - \sum_{m \in \mathcal{M}} b_m(t) \cdot H_m(t)$ in the RHS of inequality (15) is a constant in each time slot, we should maximize $\Psi_1(t)$, $\Psi_2(t)$ and $\Psi_3(t)$, as follows.

End-to-End Rate Control: At each source s_m, the admissible end-to-end rates $r_m(t)$'s are computed by solving

$$\max_{\eta_m} \quad \Psi_1(t) \quad (17)$$
$$s.t. \quad 0 \leq \eta_m(t) \leq A_{max}^{(m)}, \ \forall m \in \mathcal{M},$$

and,

$$\max_{r_m} \quad \Psi_2(t) \quad (18)$$
$$s.t. \quad 0 \leq r_m(t) \leq A_m(t), \ \forall m \in \mathcal{M}.$$

(17) is a convex optimization problem, and (18) is linear maximization. We can compute the optimal solutions of $\eta_m(t)$ and $r_m(t)$ as follows:

$$\eta_m(t) = \max\{\min\{U'^{-1}(\frac{Y_m(t)}{V}), A_{max}^{(m)}\}, 0\}, \quad (19)$$

$$r_m(t) = \begin{cases} A_m(t) & Y_m(t) - Q_{s_m}^{(m)}(t) > 0 \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

Here $U'^{-1}(\cdot)$ is the inverse function of $U'(\cdot)$, the first order derivative of function $U(\cdot)$.

Joint Routing, Channel Allocation and Pricing: In the secondary user network, routing, channel allocation and pricing decisions, $\mu_{ij}^{(m)}(t)$, $\alpha_{ij}^{(c)}(t)$ and $z_{ij}^{(c)}(t), \forall e_{ij} \in \mathcal{E}, \forall m \in \mathcal{M}, c \in \mathcal{C}$, are made by solving

$$\max_{\mu_{ij}^{(m)}, \alpha_{ij}^{(c)}, z_{ij}^c} \quad \Psi_3(t)$$
$$s.t. \quad \text{Constraints } (2)(3)(4)(5)(6)(7)(8).$$

This problem can be simplified into a pure channel allocation problem related only to variables $\alpha_{ij}^{(c)}(t)$, as follows:

Constraint (3) implies that at most one channel is assigned to each link during each time slot, *i.e.*, $\sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \leq 1$. Constraint (2) further leads to $\sum_{m \in \mathcal{M}} \mu_{ij}^{(m)}(t) = \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \leq 1$. To maximize $\Psi_3(t)$, if channel c is allocated for transmission over link e_{ij} in t (*i.e.*, $\alpha_{ij}^{(c)}(t) = 1$ and $\alpha_{ij}^{(c')}(t) = 0, \forall c' \neq c$), a transmission of data in session $\hat{m}_{ij}^{(c)}$ should be scheduled, the session associated with the largest weight $W_{ij}^{(m)}(t)$ (Eqn. (16)):

$$\hat{m}_{ij}^{(c)} = \arg \max_{m \in \mathcal{M}, i \neq d_m} \{q_{s_m}^{(m)} [\frac{Q_i^{(m)}(t)}{q_i^{(m)}} - \frac{Q_j^{(m)}(t)}{q_j^{(m)}}] - H_m(t) \cdot z_{ij}^{(c)}(t)\}. \quad (21)$$

Hence, $\hat{m}_{ij}^{(c)}$ is a function of $z_{ij}^{(c)}(t)$.

Then the joint routing, channel allocation and pricing problem can be reduced to the following joint channel allocation and pricing problem:

$$\max_{\alpha_{ij}^{(c)}, z_{ij}^{(c)}} \quad \Psi_4(t) = \sum_{e_{ij} \in \mathcal{E}} \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \cdot \left[W_{ij}^{(\hat{m}_{ij}^{(c)})}(t) - V \cdot z_{ij}^{(c)}(t) \right]$$

$$s.t. \quad \text{Constraints } (3)(4)(5)(6)(7)(8).$$

Given any feasible channel allocation decisions $\boldsymbol{\alpha}(t)$ satisfying constraints (3)–(8), to maximize $\Psi_4(t)$, the term $W_{ij}^{(\hat{m}_{ij}^{(c)})}(t) - V \cdot z_{ij}^{(c)}(t)$ should be maximized at each $\alpha_{ij}^{(c)}(t) = 1$. We can decide such a price $z_{ij}^{(c)}(t)$ by

$$z_{ij}^{(c)}(t) = \arg \max_{\xi \in S_{ij}^{(c)}} \{ W_{ij}^{(\hat{m}_{ij}^{(c)})}(t) - V \cdot \xi \}.$$

Here $S_{ij}^{(c)} = F_{ij}^{(c)-1}(G, I, \mathcal{C}, \boldsymbol{\alpha}(t))$ denotes the set of possible values of price $z_{ij}^{(c)}(t)$ leading to the same allocation $\boldsymbol{\alpha}(t)$.

Based on the above, the joint channel allocation and pricing problem can be further reduced to a pure channel allocation problem, decided by $\alpha_{ij}^{(c)}(t)$'s only:

$$\max_{\alpha_{ij}^{(c)}} \quad \Psi_5(t) = \sum_{e_{ij} \in \mathcal{E}} \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \cdot \max_{\xi \in S_{ij}^{(c)}} \left\{ W_{ij}^{(\hat{m}_{ij}^{(c)})}(t) - V \cdot \xi \right\}$$
$$(22)$$

$$s.t. \quad \text{Constraints } (3)(4)(5)(6).$$

Given a specific primary user mechanism represented by $F_{ij}^{(c)}(\cdot)$, $\forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}$, the channel allocation decisions $\alpha_{ij}^{(c)}(t)$'s can be derived by solving the above optimization. We can then make the pricing and routing decisions as follows:

$$z_{ij}^{(c)}(t) = \begin{cases} \arg\max_{\xi \in S_{ij}^{(c)}} \{ W_{ij}^{(\hat{m}_{ij}^{(c)})}(t) - V \cdot \xi \} & \text{if } \alpha_{ij}^{(c)}(t) = 1 \\ 0 & \text{otherwise} \end{cases},$$
$$\forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}, \quad (23)$$

$$\mu_{ij}^{(m)}(t) = \begin{cases} \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) & \text{if } m = \hat{m}_{ij}^{(c)} \\ 0 & \text{otherwise} \end{cases}, \forall e_{ij} \in \mathcal{E}, m \in \mathcal{M}.$$
$$(24)$$

The sketch of our dynamic, social welfare maximization algorithm is summarized in Algorithm 1. The implication of the joint routing, channel allocation and pricing is to prioritize transmissions of data in sessions with low budget deficits (backlog of virtual queue $H_m(t)$), from a more congested node (with high buffer occupancy ratio $Q_i^{(m)}(t)/q_i^{(m)}$) to a less congested node (with low buffer occupancy ratio $Q_j^{(m)}(t)/q_j^{(m)}$), using a channel with a low leasing price.

D. Distributed Implementation with Spectrum Auctions at Primary Users

We next discuss a distributed protocol to solve the social welfare maximization problem in (10), given a spectrum auction mechanism at the primary users. Auctions are a typical category of spectrum selling mechanisms, which has been extensively studied in recent literature [6], [8], [11], [22], [23].

In a typical spectrum auction, secondary users bid for idle channels at the primary users, who may greedily allocate the channels to maximize their revenues based on the bidding prices, *i.e.*, by solving an optimization problem as follows:

$$\max_{\alpha_{ij}^{(c)}} \quad \Omega(t) = \sum_{e_{ij} \in \mathcal{E}} \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t) \quad (25)$$

$$s.t. \quad \text{Constraints } (3)(4)(5)(6),$$

Algorithm 1 Dynamic Social Welfare Maximization Algorithm in Time Slot t

Input: $Q_i^{(m)}(t)$, $Y_m(t)$, $H_m(t)$, $A_m(t)$, $A_{max}^{(m)}$, $b_m(t)$, $b_{max}^{(m)}$, V, $q_i^{(m)} (\forall i \in V_S, \forall m \in \mathcal{M})$.

Output: $r_m(t)$, $\eta_m(t)$, $\alpha_{ij}^{(c)}(t)$, $z_{ij}^{(c)}(t)$, $\mu_{ij}^{(m)}(t)$ $(\forall i \in V_S, \forall m \in \mathcal{M}, \forall c \in \mathcal{C}, \forall e_{ij} \in \mathcal{E})$.

1: **End-to-End Rate Control**: For each session $m \in \mathcal{M}$, the end-to-end rate $r_m(t)$ and auxiliary variable $\eta_m(t)$ are decided at source s_m with Eqn. (20) and (19), respectively.
2: **Joint Routing, Channel Allocation and Pricing**: For each link $e_{ij} \in \mathcal{E}$ and channel $c \in \mathcal{C}$, calculate $\hat{m}_{ij}^{(c)}$ with Eqn. (21).
   Derive channel allocation $\alpha_{ij}^{(c)}(t)$ and pricing variables $z_{ij}^{(c)}(t), \forall e_{ii} \in \mathcal{E}, c \in \mathcal{C}$, by solving problem (22) and Eqn. (23), respectively, or using our distributed Alg. 2 (in case that primary users use spectrum auctions).
   Decide routing decisions $\mu_{ij}^{(m)}(t)$, $\forall e_{ij} \in \mathcal{E}, m \in \mathcal{M}$, with Eqn. (24).
3: Update queues $Q_i^{(m)}(t+1)$, $Y_m(t+1)$, and $H_m(t+1)$ based on queuing law (1), (12), and (14), respectively.

---

where the constraints ensure collision-free channel allocation. $z_{ij}^{(c)}(t)$'s are known values to primary users as the bidding prices. $\alpha_{ij}^{(c)}(t) \cdot z_{ij}^{(c)}(t)$ indicates the revenue gained by a primary user by leasing channel c to link e_{ij}. We next show how the secondary users set the bidding prices, *i.e.*, $\mathbf{Z}(t) = \{z_{ij}^{(c)}(t) | e_{ij} \in \mathcal{E}, c \in \mathcal{C}\}$, in order to get their desired channel allocation decisions, *i.e.*, $\boldsymbol{\alpha}(t) = \{\alpha_{ij}^{(c)}(t) | e_{ij} \in \mathcal{E}, c \in \mathcal{C}\}$.

When the primary users apply the above auction mechanism to decide collision-free channel allocation $\alpha_{ij}^{(c)}(t)$'s with given prices $z_{ij}^{(c)}(t)$'s, the following observations hold: (1) $\alpha_{ij}^{(c)}(t) = 0$ if $z_{ij}^{(c)}(t) = 0$; (2) if setting $\alpha_{ij}^{(c)}(t) = 1$ when $z_{ij}^{(c)}(t) \in [z_{min}, z_{max}]$ and 0 otherwise, generates collision-free channel allocation decisions $\boldsymbol{\alpha}(t)$ that satisfy constraints (3)-(6), then $\Omega(t)$ is maximized at that $\boldsymbol{\alpha}(t)$. The rationale is, for the collision-free channel allocation $\boldsymbol{\alpha}(t)$, we know that i) changing any $\alpha_{ij}^{(c)}(t)$ from 1 to 0 decreases $\Omega(t)$, since we already have $z_{ij}^{(c)}(t) \in [z_{min}, z_{max}]$ and $z_{min} > 0$, and ii) changing any $\alpha_{ij}^{(c)}(t)$ from 0 to 1 does not change $\Omega(t)$, since such an $\alpha_{ij}^{(c)}(t)$ corresponds to $z_{ij}^{(c)}(t) = 0$.

Hence, if a set of collision-free channel allocation decisions $\boldsymbol{\alpha}(t)$ are expected, the secondary users just need to set $z_{ij}^{(c)}(t)$'s as follows, in order to achieve this $\boldsymbol{\alpha}(t)$ as the result of spectrum auction in (25): $z_{ij}^{(c)}(t)$ can be set to any value in $[z_{min}, z_{max}]$ to achieve $\alpha_{ij}^{(c)}(t) = 1$, and set to 0 to achieve $\alpha_{ij}^{(c)}(t) = 0$. Therefore, in case of this auction mechanism, the set of possible values for $z_{ij}^{(c)}(t)$ given $\alpha_{ij}^{(c)}(t) = 1$ is $F_{ij}^{(c)-1}(G, I, \mathcal{C}, \boldsymbol{\alpha}(t)) = [z_{min}, z_{max}]$.

Now consider the channel pricing equation (23) in the generic Algorithm 1. When an auction mechanism is used at the primary users, we have:

$$z_{ij}^{(c)}(t) = \begin{cases} z_{min} & \text{if } \alpha_{ij}^{(c)}(t) \text{ is expected as } 1 \\ 0 & \text{otherwise} \end{cases}, \forall e_{ij} \in \mathcal{E}, c \in \mathcal{C},$$
$$(26)$$

since now we have $S_{ij}^{(c)} = [z_{min}, z_{max}]$ if $\alpha_{ij}^{(c)}(t) = 1$, and

$W_{ij}^{(\hat{m}_{ij}^{(c)})}(t) - V \cdot \xi$ is decreasing with the increase of ξ. Hence, the channel allocation problem in (22) to decide the secondary users' expected $\boldsymbol{\alpha}(t)$, in case of the spectrum auction mechanism at the primary users, is simplified to:

$$\max_{z_{ij}^{(c)}} \quad \Psi_6(t) = \sum_{e_{ij} \in \mathcal{E}} \sum_{c \in \mathcal{C}} \alpha_{ij}^{(c)}(t) \cdot (W_{ij}^{(\hat{m}_{ij})}(t) - V \cdot z_{min}) \quad (27)$$

$$s.t. \quad \text{Constraints } (3)(4)(5)(6),$$

with

$$\hat{m}_{ij} = \arg \max_{m \in \mathcal{M}, i \neq d_m} \{ q_{s_m}^{(m)} [\frac{Q_i^{(m)}(t)}{q_i^{(m)}} - \frac{Q_j^{(m)}(t)}{q_j^{(m)}}] - H_m(t) \cdot z_{min} \}. \quad (28)$$

The resulting channel allocation problem (27) is a 0-1 integer program. A centralized solution with $(1-\delta)$-optimality can be obtained using the branch-and-bound method [12], where $\delta \in (0,1)$ is a pre-defined solution accuracy. The bidding prices $z_{ij}^{(c)}(t), \forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}$ are then determined by Eqn. (26) with the expected $\boldsymbol{\alpha}(t)$, and proposed to primary users. A nice property produced in case of this specific primary user auction mechanism is that, the expected channel allocations $\boldsymbol{\alpha}(t)$ based on the solution to problem (27) is the same as that to the spectrum auction problem (25) at the primary users, based on the above discussion and the fact that the expected $\boldsymbol{\alpha}(t)$ are collision-free. That is, using spectrum prices by Eqn. (26) and solving problem (27), spectrum auctions at the primary users lead to exactly the same channel allocation decisions, as desired by the secondary users to maximize their social welfare.

We next propose a distributed implementation to solve the channel allocation problem in (27) and determine the bidding prices at each secondary user, as given in Algorithm 2. It is worth noting that, for Algorithm 1, i) the end-to-end rate control is already a distributed solution, since each auxiliary variable $\eta_m(t)$ and rate control variable $r_m(t)$ of session $m \in \mathcal{M}$ can be derived with Eqn. (19) and (20), respectively, on source node s_m based on its local queues $Y_m(t)$ and $Q_{s_m}^{(m)}(t)$; ii) each channel pricing variable $z_{ij}^{(c)}(t)$, $\forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}$, can be assigned with Eqn. (26) if the expected value of $\alpha_{ij}^{(c)}(t)$ is given; and iii) for each link $e_{ij} \in \mathcal{E}$, each routing variable $\mu_{ij}^{(m)}(t)$, $\forall m \in \mathcal{M}$ can be decided in a distributed fashion with Eqn. (24) based on packet queue backlogs on node i and j, and budget deficit of session m, if the expected channel allocation decisions $\alpha_{ij}^{(c)}(t)$ on each channel $c \in \mathcal{C}$ are known. Thus, if the channel allocation problem in (27) is solved in a decentralized fashion, the entire dynamic algorithm with joint rate control, routing, and channel allocation and pricing has a distributed implementation.

In the distributed channel allocation and pricing protocol, we refer to each link $e_{ij} \in \mathcal{E}$ as a "local link" of user i, and each link e_{kl} as an "interfering link" of link e_{ij} if we have either of $(e_{ij}, e_{kl}) \in I$, $k = j$, $l = i$ or $l = j$. The sender of an interfering link of e_{ij} is an "interferer" of link e_{ij}.

Each node i maintains an available channel set C_i over time. The weight of each local link e_{ij} is calculated as $w_{ij} = W_{ij}^{(\hat{m}_{ij})}(t) - V \cdot z_{min}$ based on Eqn. (16) and (28).

Each secondary user i greedily bids for one available channel, randomly selected from the commonly available channel

set of node i and node j, with price z_{min} for a link e_{ij} satisfying the three conditions in step 2 of Algorithm 2.

After channel c receiving a bid for link e_{ij}, each interferer of link e_{ij} is informed, which will exclude the channel from their available sets and further propagate the information. The bidding price $z_{ij}^{(c)}(t)$ for channel c over link e_{ij} is sent to the primary users for spectrum auction. The algorithm ends here at node i and j for the current time slot, while each non-scheduled node will continue with the above until either scheduled or with empty available channel set for each of its local links.

We will show in Sec. VI that the social welfare of the distributed protocol is close to that achieved by the centralized branch-and-bound channel allocation method.

Algorithm 2 Distributed Channel Allocation and Pricing Protocol at Secondary User i in Time Slot t

---

Input: C_i, $Q_i^{(m)}(t)$, $H_m(t)$, $q_{s_m}^{(m)}$, and $q_i^{(m)}$ ($\forall m \in \mathcal{M}$)
Output: $z_{ij}^{(c)}(t)$ and expected $\alpha_{ij}^{(c)}(t)$ ($\forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}$)

1: **Initialization**
   - Initialize channel pricing and expected channel allocation variables $z_{ij}^{(c)}(t) \leftarrow 0$, $\alpha_{ij}^{(c)}(t) \leftarrow 0, \forall e_{ij} \in \mathcal{E}, c \in \mathcal{C}$;
   - Obtain available channel set C_j and queue sizes $Q_j^{(m)}(t)$, $\forall m \in \mathcal{M}$ from receiver j of each local e_{ij};
   - Calculate weight $w_{ij} = W_{ij}^{(\hat{m}_{ij})}(t) - V \cdot z_{min}$ based on Eqn. (16) and (28); propagate w_{ij} and commonly available channel set $C_i \bigcap C_j$ for each local e_{ij} to its interferers;

2: **Channel pricing**: If local link e_{ij} satisfies: (1) $C_i \bigcap C_j$ is not empty; (2) $w_{ij} > 0$ and is largest among w_{lk} on its interfering links e_{lk}; (3) w_{ij} is the largest among weights on all those local links at node i, which have the largest weights among their respective interfering links as well.
   Randomly select $c \in C_i \bigcap C_j$ and bid it for e_{ij} by setting $z_{ij}^{(c)}(t) = z_{min}$ and $\alpha_{ij}^{(c)}(t) = 1$; inform each interferer of e_{ij} about bidding and expected channel allocation, send bidding price $z_{ij}^{(c)}(t)$ to primary users, and end the algorithm here and that at node j.

3: **Information update**: Upon receiving a channel bidding decision and expected channel allocation, update available channel sets for local links and convey updates to interferers of local links.

4: The algorithm ends if either the available channel set is empty for each local link or node i is scheduled as a receiver by some other node; Otherwise, go to step 2.

On Practical implementation. Similar to common practices [2], [3], [14], a Common Control Channel (CCC) defined on an unlicensed spectrum available to all secondary users is utilized to propagate the control messages, *e.g.*, queue backlogs, common available channel sets and weights for each link, and bidding decisions. The interference relationship between link pairs are also derived by sending pilot bits and detecting collisions over CCC.

Another concern is that Algorithm 1 makes joint routing, channel allocation and pricing decisions based on the accurate lengths of virtual queues $H_m(t)$, $\forall m \in \mathcal{M}$. In a distributed implementation of the algorithm, $H_m(t)$ is maintained at source s_m, and may not be immediately available at each relay node of session m. Therefore, it is likely that relay nodes have to make decisions in time slot t based on delayed queue

backlog information $H_m(t-T)$ with $T > 0$. We will show in Sec. V that even in this case, our algorithm can still achieve a social welfare arbitrarily close to the offline optimum when $V \to \infty$.

V. PERFORMANCE ANALYSIS

We now analyze the algorithm presented in Algorithm 1. We prove that our algorithm achieves approximately optimal social welfare, while guaranteeing finitely bounded buffer sizes at all nodes without overflow.

Theorem 2 (Finite Buffer without Overflow): For each data session $m \in \mathcal{M}$, define

$$Y_{max}^{(m)} \triangleq VU'(0) + A_{max}^{(m)}, \quad q_{s_m}^{(m)} \triangleq VU'(0) + 2A_{max}^{(m)} + 1,$$

$$H_{max}^{(m)} \triangleq (VU'(0) + 2A_{max}^{(m)} + 1)/z_{min} + \frac{|V_S|}{2} \cdot z_{max}.$$

Queue sizes $Y_m(t)$, $Q_{s_m}^{(m)}(t)$, $H_m(t)$, and $Q_i^{(m)}(t)$ ($\forall i \neq s_m$, $i \neq d_m$) are upper-bounded by buffer sizes $Y_{max}^{(m)}, q_{s_m}^{(m)}, H_{max}^{(m)}$, and any given non-negative $q_i^{(m)}$, respectively, without buffer overflow, *i.e.*, $Y_m(t) \leq Y_{max}^{(m)}$, $Q_{s_m}^{(m)}(t) \leq q_{s_m}^{(m)}$, $H_m(t) \leq H_{max}^{(m)}$, and $Q_i^{(m)}(t) \leq q_i^{(m)}$ in each time slot t.

The theorem can be proven by induction. Detailed proofs are included in our technical report [15].

Theorem 3 (Social Welfare Optimality): The social welfare achieved by Algorithm 1 is within a constant gap $\frac{B}{V}$ from the optimum social welfare ϕ^*—the offline optimum derived by an algorithm with complete information of the system over a long run, *i.e.*,

$$\phi \geq \phi^* - \frac{B}{V},$$

and all queues in the network are stable as in Theorem 2. Here B and V are constants defined in Sec. IV.

The detailed proof is given in our technical report [15].

Theorem 4 (Finite Buffer with Delayed Information): Suppose that the maximum delay for information of virtual queue length $H_m(t)$ at source s_m of session $m, \forall m \in \mathcal{M}$, to propagate to each secondary user, is T time slots. For each session $m \in \mathcal{M}$, define

$$H_{max}^{(m)'} \triangleq (VU'(0) + 2A_{max}^{(m)} + 1)/z_{min} + (T+1) \cdot \frac{|V_S|}{2} \cdot z_{max}.$$

Queue sizes $Y_m(t)$, $Q_{s_m}^{(m)}(t)$, $H_m(t)$, and $Q_i^{(m)}(t)$ (for all $i \neq s_m$ and $i \neq d_m$) are upper-bounded by buffer sizes $Y_{max}^{(m)}$, $q_{s_m}^{(m)}, H_{max}^{(m)'}$, and any given non-negative $q_i^{(m)}$, respectively, without buffer overflow, *i.e.*, $Y_m(t) \leq Y_{max}^{(m)}$, $Q_{s_m}^{(m)}(t) \leq q_{s_m}^{(m)}$, $H_m(t) \leq H_{max}^{(m)'}$, and $Q_i^{(m)}(t) \leq q_i^{(m)}$ in each time slot t.

The theorem can also be proven by induction, with detailed proofs in our technical report [15].

Theorem 5 (Optimality with Delayed Information): Suppose that the maximum delay for information of virtual queue length $H_m(t)$ at source s_m of session $m, \forall m \in \mathcal{M}$, to arrive at each secondary user, is T time slots. The social welfare achieved by Algorithm 1 is within a constant gap $\frac{B + B_2 \cdot T}{V}$ from the optimum social welfare ϕ^*, *i.e.*,

$$\phi \geq \phi^* - \frac{B + B_2 \cdot T}{V},$$

and all queues in the network are stable as in Theorem 4, where $B_2 = \max_{m \in \mathcal{M}}\{b_{max}^{(m)}\} \cdot \frac{|V_S|}{2} \cdot z_{max} + (\frac{|V_S|}{2})^2 \cdot (z_{max})^2$ is a constant. B and V are constants defined in Sec. IV.

The proof is similar to that of Theorem 3, details can be found in the technical report [15].

Corollary 1: The social welfare achieved with Algorithm 1 can be arbitrarily close to the offline maximum ϕ^*, when $V \to \infty$ and $q_i^{(m)} \propto V$, $\forall m \in \mathcal{M}, i \in V_S, i \neq d_m$.

Remarks: Since B/V and $(B + B_2 \cdot T)/V$ are inversely proportional to V while $q_{s_m}^{(m)}$, $q_i^{(m)}$ (with Corollary 1), $Y_{max}^{(m)}$, $H_{max}^{(m)}$ and $H_{max}^{(m)'}$ are proportional to V ($\forall m \in \mathcal{M}, i \neq d_m$), a tradeoff exists between (i) social welfare optimality, *i.e.*, constant gap B/V or $(B + B_2 \cdot T)/V$, and (ii) buffer usages, *i.e.*, $Y_{max}^{(m)}$, $q_{s_m}^{(m)}$ and $q_i^{(m)}$ ($\forall m \in \mathcal{M}, i \neq d_m$), and allowed worst-case budget deficits, *i.e.*, $H_{max}^{(m)}$ or $H_{max}^{(m)'}$.

VI. EMPIRICAL STUDIES

We evaluate our proposed algorithms through simulations, incorporating a spectrum auction as the primary users' mechanism. The social welfare, achieved by the distributed implementation, is compared to that of a centralized algorithm following Algorithm 1 (proven to be close to the offline optimum) under various patterns of data arrival and budget provision. We also examine the tradeoff between (i) social welfare, and (ii) buffer sizes and budget deficits, by varying the value of V.

A. Simulation Setup

We experiment with networks of 20 secondary users and 4 primary users *uniformly* randomly distributed in a square of $10,000$ m^2. There are 5 unicast sessions, with randomly chosen sources and destinations among the secondary users. The maximum data arrival and budget provision rates for each session, A_{max} and b_{max}, are chosen from $\{0.5, 1.0, 1.5, 2.0\}$ and $\{0.1, 0.15, 0.2, 0.25\}$, respectively. The idle probability of primary users follows a uniform distribution between 0 and 1, with an expectation of 0.7 in each time slot. The protocol interference model [9] is employed, in which a transmission is successful if a receiver is within the transmission range of its sender and outside of the interference range of other concurrent senders. Each primary or secondary node has a transmission range and an interference range of 30 meters. The minimum price for spectrum leasing z_{min} is set to 0.1. Parameter V is chosen among 500, 1000 and 2000. In the centralized Algorithm 1, the 0-1 program (27) is solved by the `glpk` tool kit [1] in each time slot. The throughput utility function is $U(x) = \log(x + 1)$. All our results presented are the average of 1000 trials, each of which runs the algorithms for a duration of $T = [1, 10^6]$ (time slots).

B. Social Welfare with Centralized and Distributed Algorithms

We compare the social welfare achieved by the centralized and distributed algorithms in Algorithm 1, and Algorithm 2, respectively. V is set to 2000. In Fig. 1(a), we fix the maximum budget provision rate b_{max}, but vary the maximum data arrival rates A_{max}; in Fig. 1(b), the maximum data arrival rate is fixed while the maximum budget arrival rate varies. The buffer sizes are set following $q_i^{(m)} = V/10$, $\forall i \in V_S, m \in \mathcal{M}, i \neq s_m, i \neq d_m$. We can observe that the social welfare achieved

with the distributed algorithm is close to that achieved by the centralized algorithm under each setting.
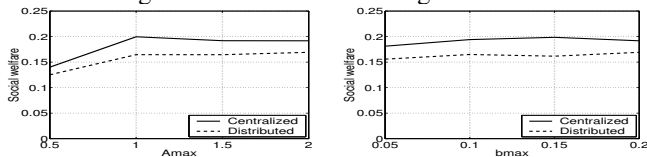
(a) $V = 2000$, $b_{max} = 0.25$. (b) $V = 2000$, $A_{max} = 2.0$.

Fig. 1. Social welfare: centralized algorithm vs. distributed algorithm.

C. Impact of V

According to Corollary 1 in Sec. V, different values of V achieve different tradeoffs between the social welfare and the buffer usage at the nodes. With the increase of V, the social welfare by our algorithm can be arbitrarily close to the offline maximum, while the buffer size at each secondary user is proportional to V.
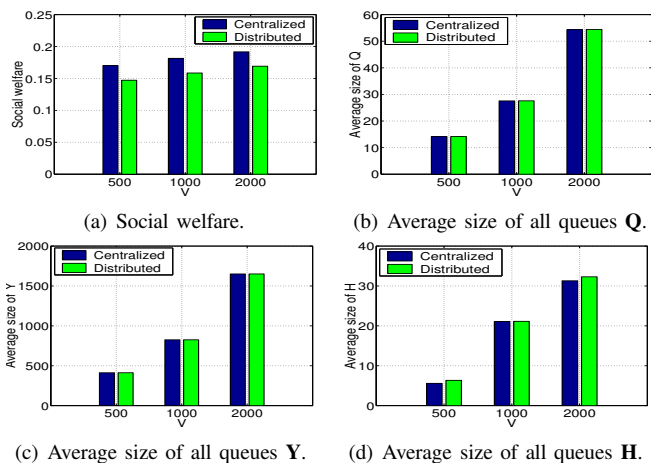

(a) Social welfare. (b) Average size of all queues $\mathbf{Q}$.

(c) Average size of all queues $\mathbf{Y}$. (d) Average size of all queues $\mathbf{H}$.

Fig. 2. Tradeoff between social welfare and buffer usage with different V's.

In the experiments in Fig. 2, A_{max} and b_{max} are set to 2.0 and 0.25, respectively. The social welfare increases with the increase of V, and stabilizes when V is larger than 1000. On the other hand, the average size of all packet queues $\mathbf{Q} = \{Q_i^{(m)}, \forall m \in \mathcal{M}, i \in \mathcal{V}_\mathcal{S}, i \neq d_m\}$, the average size of all virtual queues for budget deficits $\mathbf{H} = \{H_m, \forall m \in \mathcal{M}\}$, and the average size of all virtual queues $\mathbf{Y} = \{Y_m, \forall m \in \mathcal{M}\}$ increase proportionally with the increase of V in Fig. 2(b)-2(d). All these validate our analysis in Sec. V.

VII. CONCLUSION AND REMARKS

This paper proposes a social welfare maximization framework for multi-session multi-hop data communication in secondary user networks where the secondary users have practical budget constraints for spectrum purchase. Given any spectrum selling mechanism of the primary users, a dynamic, joint rate control, routing, channel allocation and pricing protocol is derived for secondary users to make socially optimal decisions of spectrum purchase and data delivery at any given time. We also design a practical distributed implementation in the case where spectrum auction is the assumed mechanism of the primary users. Rigorous theoretical analyses demonstrate that the proposed dynamic protocol, regardless of whether the information from other nodes is timely or delayed, can achieve a time-averaged social welfare (throughput minus cost)

among all secondary users that is arbitrarily close to the offline optimum. The analyses also show nice guarantees of finite buffer sizes for all queues without buffer overflow, as well as bounded budget deficits in the worst cases. All these are further verified using simulations under realistic settings.

REFERENCES

[1] Glpk (gnu linear programming kit), http://www.gnu.org/s/glpk/.
[2] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty. NeXt Generation/Dynamic Spectrum Access/Cognitive Radio Wireless Networks: A Survey. *Computer Networks*, 50(13):2127 – 2159, 2006.
[3] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty. A Survey on Spectrum Management in Cognitive Radio Networks. *IEEE Communications Magazine*, 08:40–48, April 2008.
[4] P. Cramton. Spectrum auctions. *Handbook of Telecommunications Economics*, page 605639, 2002.
[5] Z. Feng and Y. Yang. Joint transport, routing and spectrum sharing optimization for wireless networks with frequency-agile radios. In *Proc. of IEEE INFOCOM'09*, 2009.
[6] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri. A general framework for wireless spectrum auctions. In *Proc. of IEEE DySPAN'07*, 2007.
[7] L. Gao, X. Wang, Y. Xu, and Q. Zhang. Spectrum trading in cognitive radio networks: A contract-theoretic modeling approach. *IEEE Journal on Selected Areas in Communications*, 29:843 – 855, Apr. 2011.
[8] A. Gopinathan, Z. Li, and C. Wu. Strategyproof auctions for balancing social welfare and fairness in secondary spectrum markets. In *Proc. of IEEE INFOCOM'11*, 2011.
[9] P. Gupta and P. Kumar. The capacity of wireless networks. *IEEE Trans. on Information Theory*, 46(2):388–404, Mar. 2000.
[10] S. Haykin. Cognitive Radio: Brain-Empowered Wireless Communications. *IEEE Journal on Selected Areas in Communications*, 23(2):201 – 220, Feburary 2005.
[11] J. Jia, Q. Zhang, Q. Zhang, and M. Liu. Revenue generation for truthful spectrum auction in dynamic spectrum access. In *Proc. of ACM MobiHoc'09*, 2009.
[12] E. L. Lawler and D. E. Wood. Branch-and-bound methods: A survey. *Operations Research*, 14:699–719, 1966.
[13] L. B. Le, E. Modiano, and N. B. Shroff. Optimal control of wireless networks with finite buffers. In *Proc. of IEEE INFOCOM'10*, 2010.
[14] H. Li, W. Huang, C. Wu, Z. Li, and F. C. M. Lau. Utility-maximizing data dissemination in socially selfish cognitive radio networks. In *Proc. of IEEE MASS'11*, 2011.
[15] H. Li, C. Wu, Z. Li, and F. C. M. Lau. Socially-optimal multi-hop secondary communication under arbitrary primary user mechanisms. Technical report, The University of Hong Kong, http://i.cs.hku.hk/~hxli/spectrum-cog.pdf, 2012.
[16] M. J. Neely. *Stochastic Network Optimization with Application to Communication and Queueing Systems*. Morgan&Claypool Publishers, 2010.
[17] M. J. Neely. Opportunistic Scheduling with Worst Case Delay Guarantees in Single and Multi-Hop Networks. In *Proc. of IEEE INFOCOM'11*, 2011.
[18] D. Niyato, E. Hossain, and Z. Han. Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: A game-theoretic modeling approach. *IEEE Trans. on Mobile Computing*, 8:1009 – 1022, Aug. 2009.
[19] G. Sharma, R. R. Mazumdar, and N. B. Shroff. On the complexity of scheduling in wireless networks. In *Proc. of ACM MOBICOM'06*, 2006.
[20] Y. Shi and Y. T. Hou. A distributed optimization algorithm for multi-hop cognitive radio networks. In *Proc. of IEEE INFOCOM'08*, 2008.
[21] D. Xue and E. Ekici. Guaranteed opportunistic scheduling in multi-hop cognitive radio networks. In *Proc. of IEEE INFOCOM'11*, 2011.
[22] X. Zhou, S. Gandhi, S. Suri, and H. Zheng. ebay in the sky: Strategyproof wireless spectrum auctions. In *Proc. of ACM MobiCom'05*, 2005.
[23] X. Zhou and H. Zheng. Trust: A general framework for truthful double spectrum auctions. In *Proc. of IEEE INFOCOM'09*, 2009.
[24] Y. Zhu, B. Li, and Z. Li. Truthful spectrum auction design for secondary networks. In *In Proc. of IEEE INFOCOM'12*, 2012.