SINGLE-VIEW RECONSTRUCTION FROM AN UNKNOWN SPHERICAL MIRROR

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ABSTRACT

This paper introduces a novel method for 3D reconstruction from spherical mirrors in a single view. Traditional reconstruction algorithms either assume both intrinsic and extrinsic parameters of the cameras being known precisely and rely on multiple view information to recover 3D objects, or use reflections of the 3D objects on spherical mirrors with known radii in a single calibrated view. It will be shown in this paper that 3D objects can be recovered up to a scale using an unknown spherical mirror in a single view. Experimental results on both synthetic and real data show the practicality of the proposed reconstruction method.

***Index Terms*—** 3D reconstruction, single view, spherical mirror

1. INTRODUCTION

Recovering the shape of an object from images has many applications in robotics, augmented reality, video games and computer vision.

Given images of a 3D scene from multiple views, a 3D point can be obtained from its images by triangulation. Single-view 3D reconstruction is possible by exploiting mirrors to provide multi-view information. In the literature, there exists a large number of work dealing with catadioptric systems which consist of both mirrors and lens [1, 2]. However, they all rely on complex calibration procedures to recover the parameters of the systems. Planar mirrors are often exploited in vision systems [3, 4, 5, 6]. Since the field of view of a planar mirror image is small, only a limited part of the object can be reconstructed. Instead, a spherical mirror can be exploited for 3D reconstruction. In [7], Nayar built a system named sphereo with a camera and two spherical mirrors. The field of view of a sphereo system is greatly enhanced and is not limited by the field of view of the camera. However, the radii and the positions of the spheres must be known to recover the 3D object. In [8], Nitschke et al. reconstructed a display from its reflection on corneas of eyes, which were assumed to be spherical. In [9], Powell et al. proposed a strategy for reconstructing the positions of point light sources. The positions of the light sources can be recovered from images of highlights on spherical mirrors with known radii. In [10], Lanman et al. constructed a catadioptric system composed of a large number of identical spherical mirrors and a perspective camera. The centers and radii of the spherical mirrors were obtained through a special calibration step. In [11], Kanbara et al. moved a spherical mirror with known radius freely in the field of view of a camera to recover the surrounding scene. By attaching a color marker around the lens of the camera, the direction of the sphere center can be determined, which is then used to reconstruct the spherical mirror with the radius of the sphere. These methods all use spherical mirrors to reconstruct 3D objects. However, they all rely on known radius and/or sphere center.

In this paper, a novel 3D reconstruction algorithm, which will reconstruct a 3D object from an unknown spherical mirror in a single view, is proposed. Unlike [11] which requires the radius of the sphere and uses additional color marker to reconstruct the spherical mirror, this paper considers a sphere with unknown radius and points out that a one-parameter family of the spheres, with all the sphere centers lying on a line joining the camera center and the true sphere center, will be recovered from the silhouette of the sphere in the image. It will then be shown that a 3D object can be reconstructed up to a scale.

The rest of this paper is organized as follows. Sect. 2 addresses the problem of 3D object reconstruction with a scaled spherical mirror in a single view. It will be shown that any sphere from the family of estimated spheres can be used for reconstruction. Experimental results will be presented in Sect. 3, followed by conclusions in Sect. 4.

2. RECONSTRUCTION USING A SPHERICAL MIRROR

Consider a pinhole camera $\mathbf{P}$ centered at $\mathbf{O}$ viewing a mirror sphere $\mathbf{S}$. It was shown in [12] that $\mathbf{S}$ can be reconstructed by determining the central axis of a right circular cone estimated from the silhouette of a sphere and the camera center. If the radius R of $\mathbf{S}$ is not known, all possible sphere centers will lie along the central axis of this cone (see Fig. 1).

Now, let a 3D point $\mathbf{Q}$ be visible from $\mathbf{P}$ through its reflection on $\mathbf{S}$. The direction $\mathbf{L}$ from the point of reflection

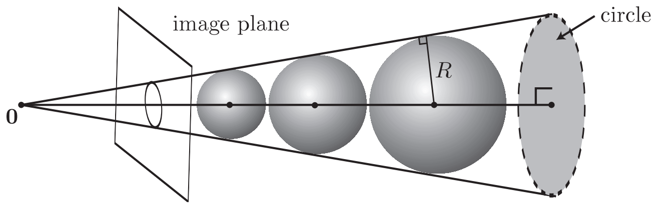


Fig. 1. The silhouette of a sphere and the camera center will define a right circular cone tangent to a family of spheres.

$\mathbf{X}$ on the spherical mirror $\mathbf{S}$ to the 3D point $\mathbf{Q}$ can be recovered from the reconstructed sphere using the law of reflection. Note that $\mathbf{L}$ is independent of the radius R used for the reconstruction of the sphere. This can be seen in Fig. 2, where a sphere is reconstructed with two different radii R_1 and R_2 resulting in two sphere centers $\mathbf{S}_{c1}$ and $\mathbf{S}_{c2}$. The points of reflection $\mathbf{X}_1$ and $\mathbf{X}_2$ project to the same point $\mathbf{q}$ in the image. The direction $\mathbf{L}_1$ from $\mathbf{X_1}$ to $\mathbf{Q}_1$ is parallel to the direction $\mathbf{L}_2$ from $\mathbf{X_2}$ to $\mathbf{Q}_2$. In general, it can be shown that there is a homothetic transformation along the axis of the cone preserving the directions of vectors.

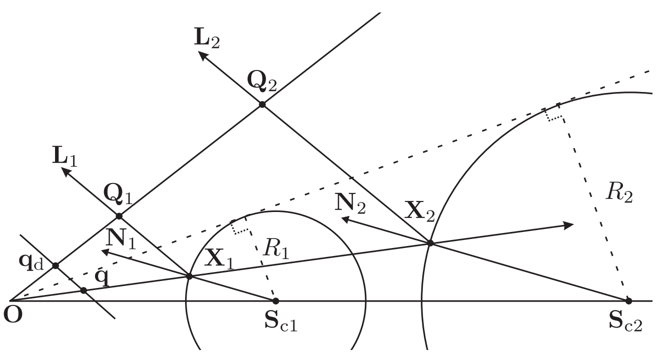


Fig. 2. Reconstruction using a direct view and a reflection on a spherical mirror.

In the following, we will use the above properties and propose a reconstruction method for two cases. For the first case (Sect. 2.1), a 3D point is visible to the camera directly and through reflection on a spherical mirror, while for the other case (Sect. 2.2), the 3D point is recovered with reflections of the point on a moving spherical mirror in two different positions. It will be shown that with unknown radius, the reconstruction result can be obtained up to an unknown scale.

2.1. Reflection and Direct View

Suppose a 3D object and a spherical mirror are placed in the view of a single calibrated camera. A 3D point $\mathbf{Q}$ on the object surface and the reflection of $\mathbf{Q}$ on the spherical mirror are both recorded by the camera, with the projections $\mathbf{q}_\mathrm{d}$ and $\mathbf{q}$ respectively (see Fig. 2). The 3D point $\mathbf{Q}$ can now be estimated from a reconstructed sphere and the two points $\mathbf{q}_\mathrm{d}$ and $\mathbf{q}$. For the sphere with radius R_1, $\mathbf{Q}_1$ is obtained as the reconstruction result triangulated using $\mathbf{L}_1$ and the visual ray of pixel $\mathbf{q}_\mathrm{d}$. For the sphere with radius R_2, $\mathbf{Q}_2$ is obtained. In the following paragraph, it will be shown that the ratio $|\overline{OQ_1}| : |\overline{OQ_2}|$ is equal to $R_1 : R_2$.

Consider the two triangles $\triangle OQ_1X_1$ and $\triangle OQ_2X_2$. As $\mathbf{L}_1 \parallel \mathbf{L}_2$, $|\overline{OQ_1}| : |\overline{OQ_2}| = |\overline{OX_1}| : |\overline{OX_2}|$. Consider now the two triangles $\triangle OX_1S_{c1}$ and $\triangle OX_2S_{c2}$. Obviously, $|\overline{OX_1}| : |\overline{OX_2}| = |\overline{X_1S_{c1}}| : |\overline{X_2S_{c2}}| = R_1 : R_2$ since $\mathbf{N}_1 \parallel \mathbf{N}_2$. It follows that $|\overline{OQ_1}| : |\overline{OQ_2}| = R_1 : R_2$. Since $\mathbf{Q}$ is an arbitrary point on the surface of the sphere, the following proposition follows.

Proposition 1 *Consider a single camera viewing an object directly visible and reflected on a spherical mirror. The object can be reconstructed up to an unknown scale using image correspondences. The scale is equal to the ratio of the radius used in recovering the sphere center to the true radius.*

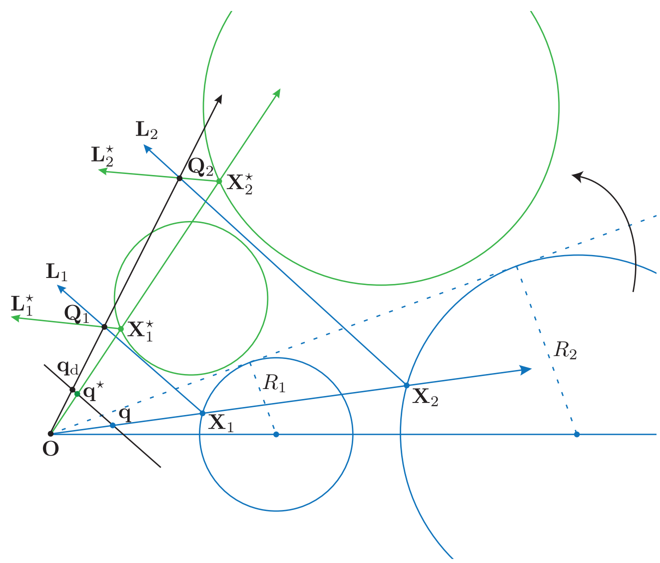


Fig. 3. Reconstruction using reflections on a moving spherical mirror.

2.2. Reflections on Multiple Spheres

Suppose the reflections of a 3D object produced on a moving spherical mirror are recorded by a single calibrated camera. Let $\mathbf{q}$ be the projection of a point $\mathbf{X}$ which reflects the point $\mathbf{Q}$ and let $\mathbf{q}^\star$ be the projection of a point $\mathbf{X}^\star$ which reflects the same point $\mathbf{Q}$ after the sphere has been moved, i.e., $\mathbf{q}$ and $\mathbf{q}^\star$ are corresponding points. In the following it will be shown that $\mathbf{Q}$ can be obtained from the point correspondence $\mathbf{q}$ and $\mathbf{q}^\star$ up to an unknown scale, if the radius of the sphere is unknown.

Suppose $\mathbf{Q}$ is directly captured by the camera with the projection $\mathbf{q}_\mathrm{d}$ in the image. A 3D point $\mathbf{T}_1$ will be obtained

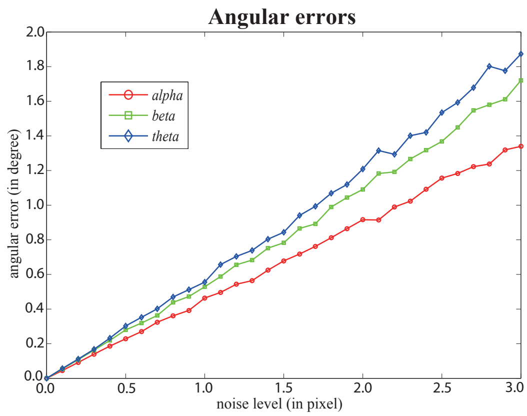


Fig. 4. Angular errors. The error increases almost linearly with the noise level.

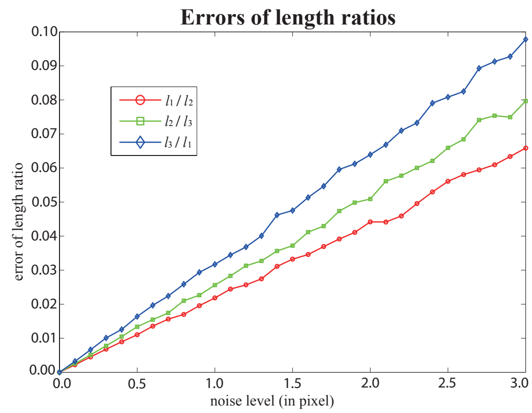


Fig. 5. Errors of length ratio. The error increases almost linearly with the noise level.

as the reconstruction result from $\mathbf{q}$ and $\mathbf{q}_{\mathrm{d}}$ if the radius of the spherical mirror is R_1. Similarity, $\mathbf{T}_1^\star$ will be reconstructed from $\mathbf{q}^\star$ and $\mathbf{q}_{\mathrm{d}}$. It follows from Sec. 2.1 that $|\overline{OT_1}| = |\overline{OT_1^\star}|$. Moreover, $\mathbf{T}_1$ and $\mathbf{T}_1^\star$ both lie on the visual ray of $\mathbf{q}_{\mathrm{d}}$. Therefore $\mathbf{T}_1$ and $\mathbf{T}_1^\star$ are the same point which both the two lines $\mathbf{L}_1$ and $\mathbf{L}_1^\star$ pass through. Since $\mathbf{L}_1$ and $\mathbf{L}_1^\star$ intersect at the point $\mathbf{Q}_1$, $\mathbf{Q}_1 = \mathbf{T}_1 = \mathbf{T}_1^\star$. Meanwhile, if the radius of the spherical mirror is R_2, then $\mathbf{T}_2$ will be reconstructed from $\mathbf{q}$ and $\mathbf{q}_{\mathrm{d}}$, and $\mathbf{T}_2^\star$ will be reconstructed from $\mathbf{q}^\star$ and $\mathbf{q}_{\mathrm{d}}$. Similarly, $\mathbf{Q}_2$, $\mathbf{T}_2$ and $\mathbf{T}_2^\star$ are the same point. It follows that $|\overline{OQ_1}| : |\overline{OQ_2}| = |\overline{OT_1}| : |\overline{OT_2}| = R_1 : R_2$. Since $\mathbf{Q}$ is an arbitrary point on the surface of the sphere, the following proposition follows.

Proposition 2 *Suppose the reflections of a 3D object produced on a moving spherical mirror are recorded by a single calibrated camera. The object can be reconstructed up to an unknown scale using correspondences of reflections in the image. The scale is equal to the ratio of the radius used in recovering the location of the sphere center to the true radius.*

3. EXPERIMENT

Experiments were carried out using both synthetic and real data. The details are presented in the following subsections.

3.1. Synthetic Data

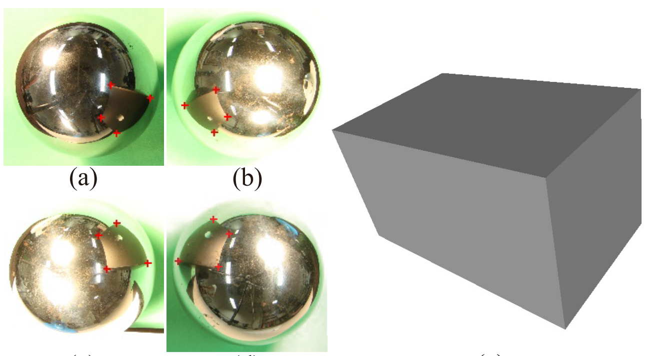
In the synthetic experiment, the reflections of four 3D points $\mathbf{X}_0$, $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$ on a moving spherical mirror in two different positions were captured by a synthetic camera. The four points had the following relations: $|\overline{X_0X_1}| = |\overline{X_0X_2}| = |\overline{X_0X_3}|$ and $\angle X_1X_0X_2 = \angle X_2X_0X_3 = \angle X_3X_0X_1 = 90°$. The images of the sphere were obtained by projecting the sphere onto the image plane. The reflection of a 3D point $\mathbf{X}(\mathbf{X} \in \{\mathbf{X}_0, \mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3\})$ on the spherical mirror was obtained with the close-form solution proposed in [13], which is then projected on the image plane.

Uniformly distributed random noise was added to the locations of 2D projections of the reflections on the sphere and the conic images of the sphere. Noise was added directly to the pixel coordinates of the locations of the reflections in the image and sampled conic points. A conic was then fitted to these noisy conic points using SVD [14].

Experiments on synthetic data with noise levels ranging from 0.0 to 3.0 pixels were carried out. For each noise level, 1000 independent trials were performed to estimate the positions of $\mathbf{X}$. The radius of the sphere was set with a random value. Denote the length of the line segment $\overline{X_0X_1}$ as l_1. Similarity, denote $|\overline{X_0X_2}| = l_2$ and $|\overline{X_0X_3}| = l_3$. Set $\angle X_1X_0X_2 = alpha$, $\angle X_2X_0X_3 = beta$ and $\angle X_3X_0X_1 = theta$. After $\mathbf{X}$ was reconstructed, error of the ratios l_1/l_2, l_2/l_3, l_3/l_1 and errors of the angles $alpha$, $beta$, $theta$ were estimated. Fig. 4 shows root mean square error of the angles (in degrees) against the noise level (in pixels). It can be seen that the error increases almost linearly with the noise level. For a noise level of 1.0 pixel, the angular errors for $alpha$, $beta$, $theta$ are all less than $0.6°$. Fig. 5 shows a plot of errors of the three ratios against the noise level (in pixels). The error increases linearly with the noise level. For a noise level of 1.0 pixel, the average errors of ratios are less than 0.04.

3.2. Real Data

In the real experiment, reflections of a rectangular box on a moving spherical mirror with the radius $3.15cm$ were captured by a camera. The intrinsic parameters of the camera were obtained using [15]. Edge points on the contours of the sphere in the images were marked by hand and conics were then fitted to these edge points using SVD. Scaled reconstruction of a sphere was achieved using the method in [12] with the radius $7cm$. The reflection correspondences of corners in different images were marked and matched manually. Four corners of the box were reconstructed using Fig. 6(a,c), while the other four corners using Fig. 6(b,d). In order to verify that the reconstruction result is up to a scale equal to the ratio of the radius used in recovering the location of the sphere center to the true radius, 24 angles and 12 length ratios of the reconstructed length to the ground truth around eight corners were measured. The ground truth of the angle is $90°$ and the ratio is 2.22. The root mean square error of the angles and the length ratios are $1.24°$ and 0.042 respectively. Around a certain corner, three angles are $91.2°$, $90.1°$, and $88.7°$, and three ratios are 2.263, 2.218 and 2.218. The reconstruction result is shown in Fig. 6(e).



(a) (b) (c) (d) (e)

Fig. 6. Reconstruction result of a box.

4. CONCLUSIONS AND FUTURE WORK

This paper addressed the problem of recovering a 3D object using an unknown spherical mirror in a single view. It has been shown in this paper that a scaled reconstruction can be obtained from either a reflection of the object on the spherical mirror and a direct image of the object or reflections on a moving spherical mirror. The reconstruction result is scaled by the unknown radius of the sphere. Experiments on both synthetic and real data show promising reconstruction results. In the future, the proposed method will be extended to obtain a dense reconstruction of a 3D model.

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